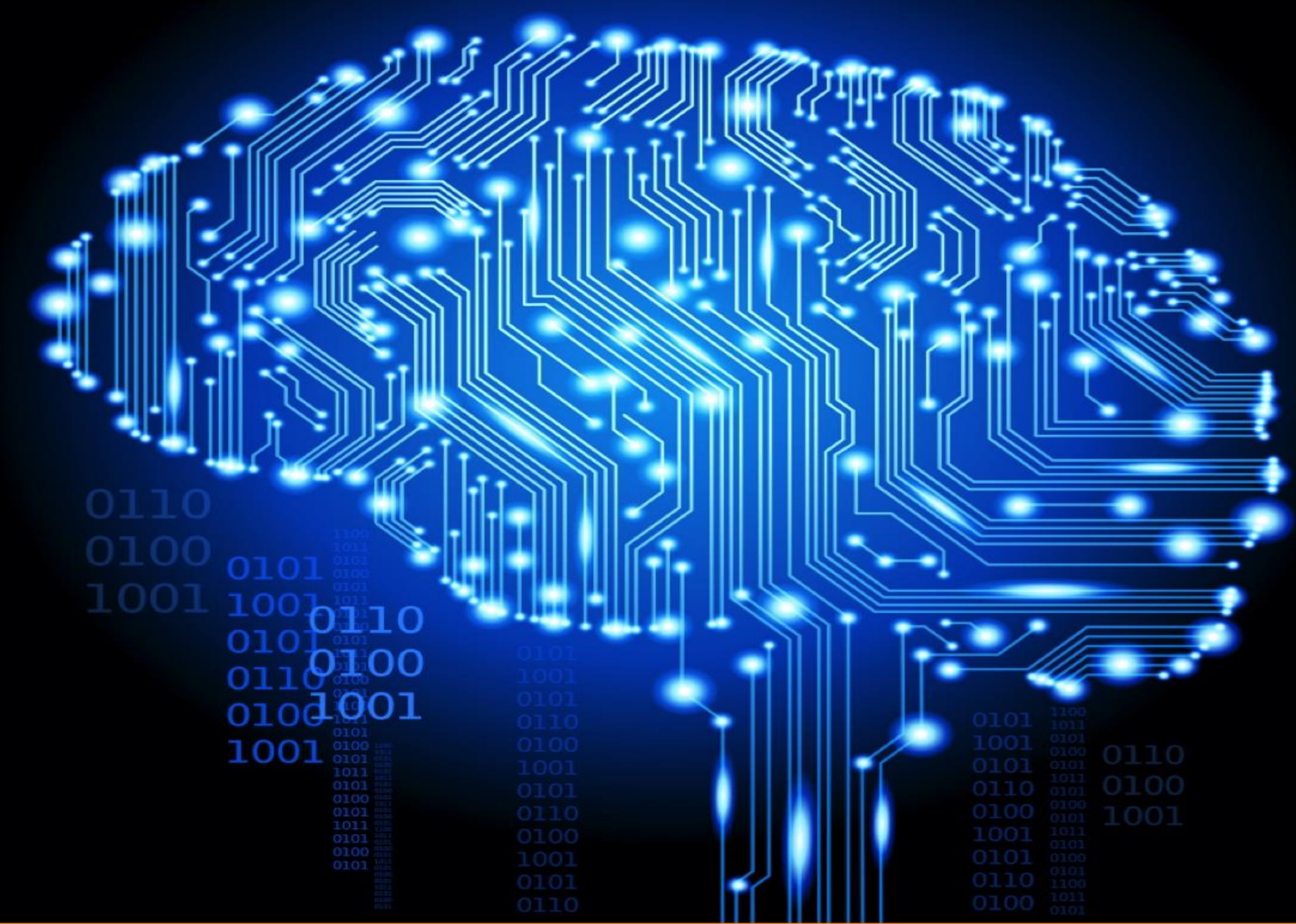


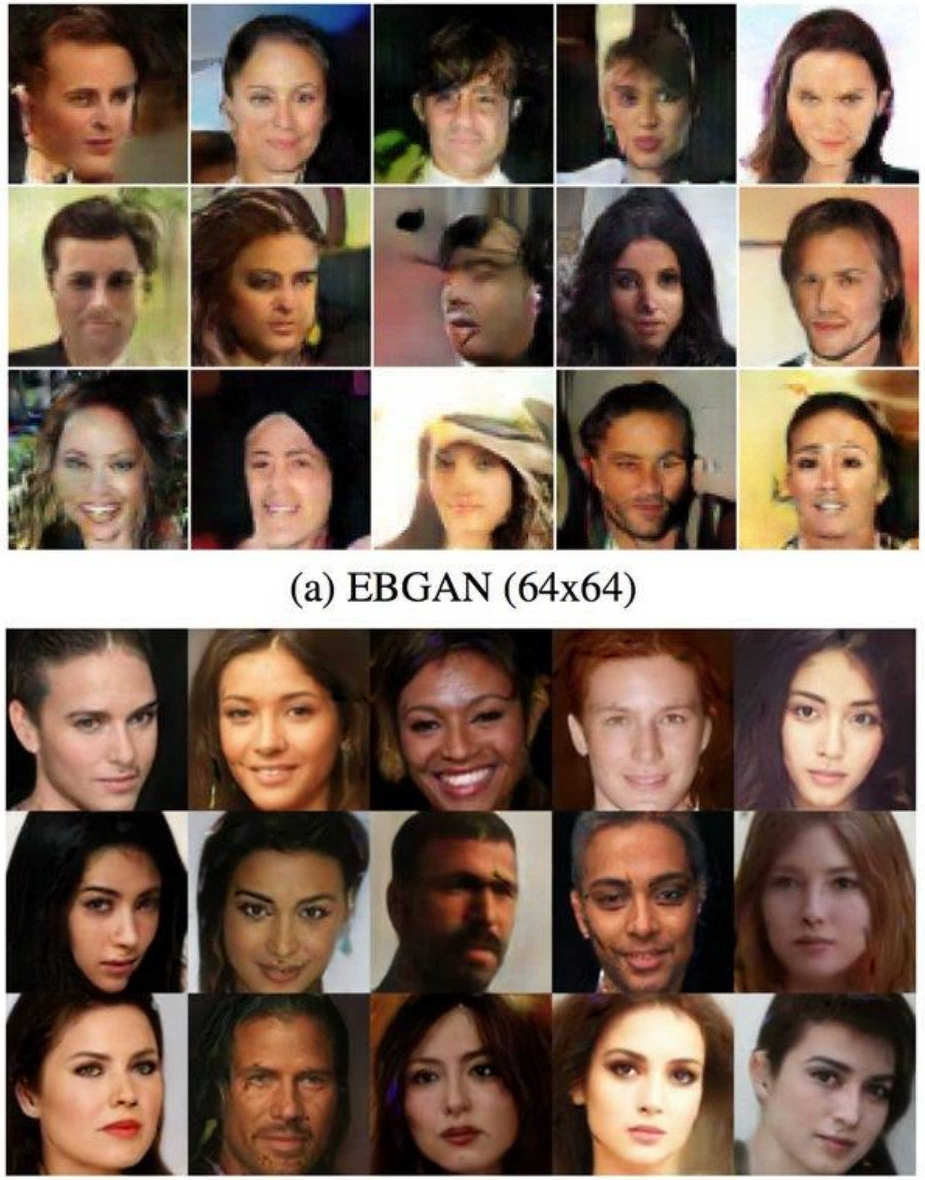
Lecture 8: Implicit Generative Models (GANs)

Efstratios Gavves

Lecture overview

- Gentle intro to generative models
- Generative Adversarial Networks
- Variants of Generative Adversarial Networks

Generative models



(a) EBGAN (64x64)

(b) Our results (128x128)

Types of Learning

- Generative modelling
  - Learn the joint pdf: $p(x, y)$
  - Model the world → Perform tasks, *e.g.* use Bayes rule to classify: $p(y|x)$
  - Naïve Bayes, Variational Autoencoders, GANs
- Discriminative modelling
  - Learn the conditional pdf: $p(y|x)$
  - Task-oriented
  - E.g., Logistic Regression, SVM

Types of Learning

- What to pick?
  - V. Vapnik: *"One should solve the [classification] problem directly and never solve a more general [and harder] problem as an intermediate step."*
- Typically, discriminative models are selected to do **the** job
- Generative models give us more theoretical guarantees that the model is going to work as intended

Why generative modeling?

Why generative modeling?

- Act as a regularizer in discriminative learning
  - Discriminative learning often too goal-oriented
  - Overfitting to the observations
- Semi-supervised learning
  - Missing data
- Simulating “possible futures” for Reinforcement Learning
- Data-driven generation/sampling/simulation

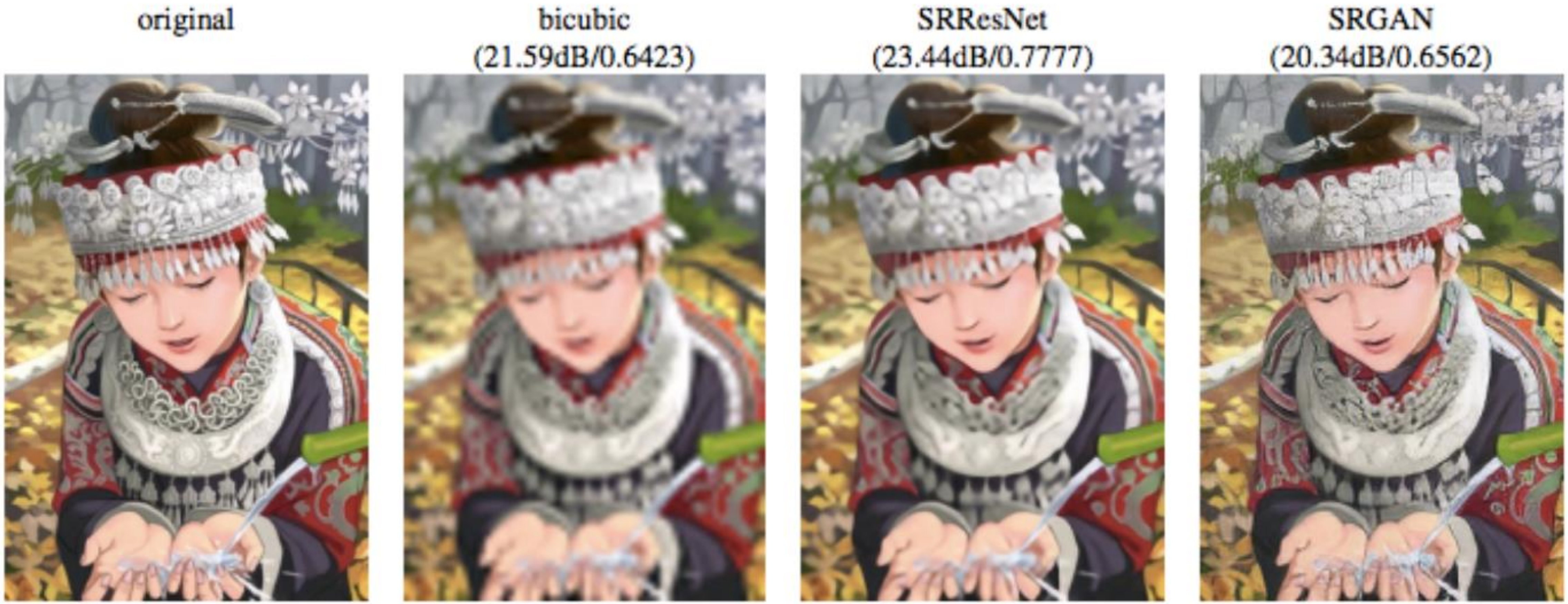
Applications: Image Generation

(a) Generated by LSGANs.

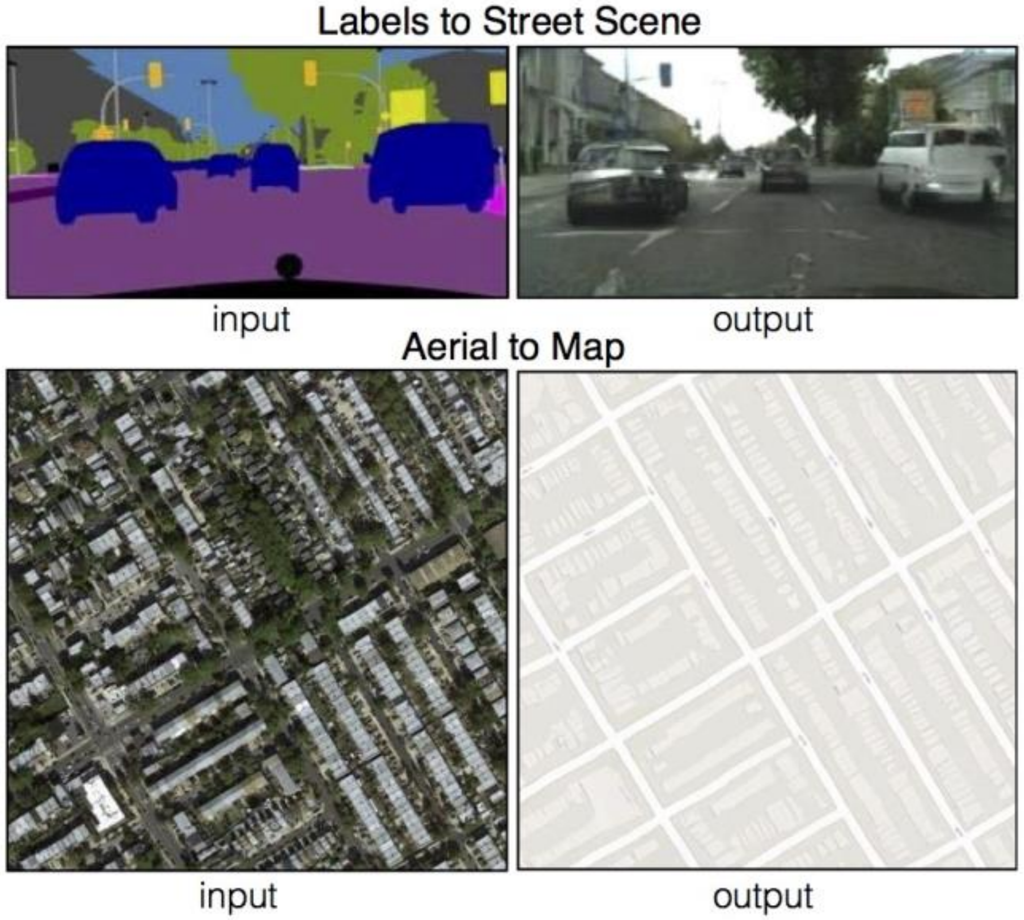
(b) Generated by DCGANs (Reported in [13]).

Figure 5: Generated images on LSUN-bedroom.

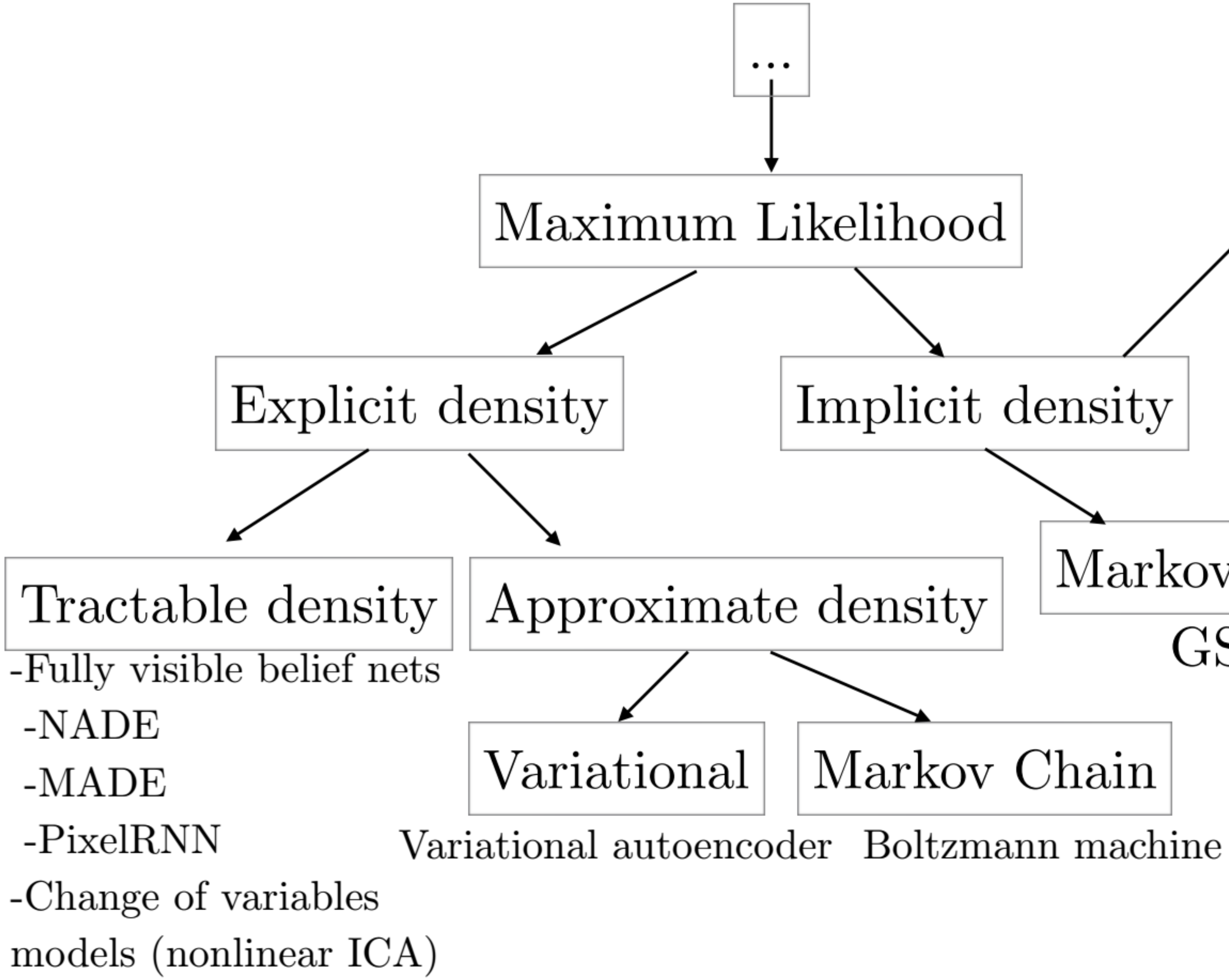
Applications: Super-resolution



Applications: Cross-model translation



A map of generative models

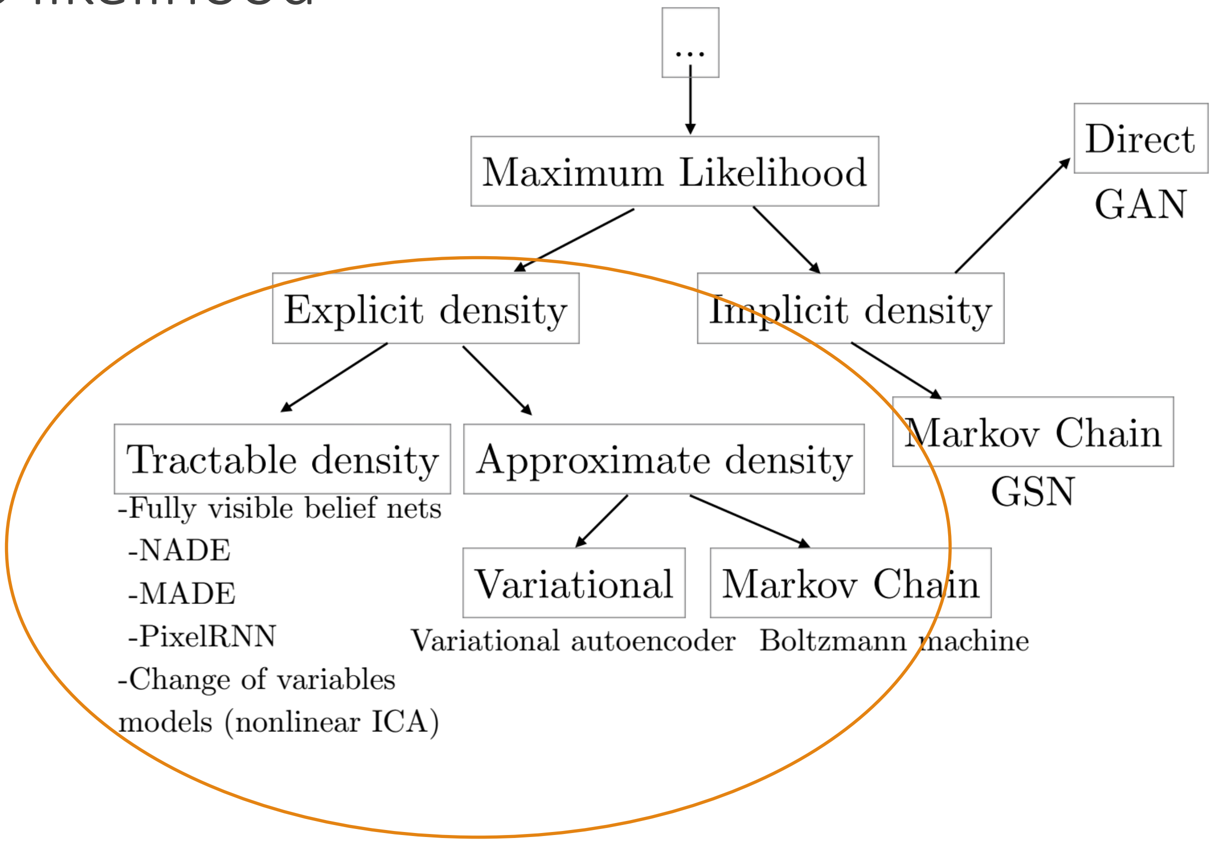


Explicit density models

- Plug in the model density function to likelihood
- Then maximize the likelihood

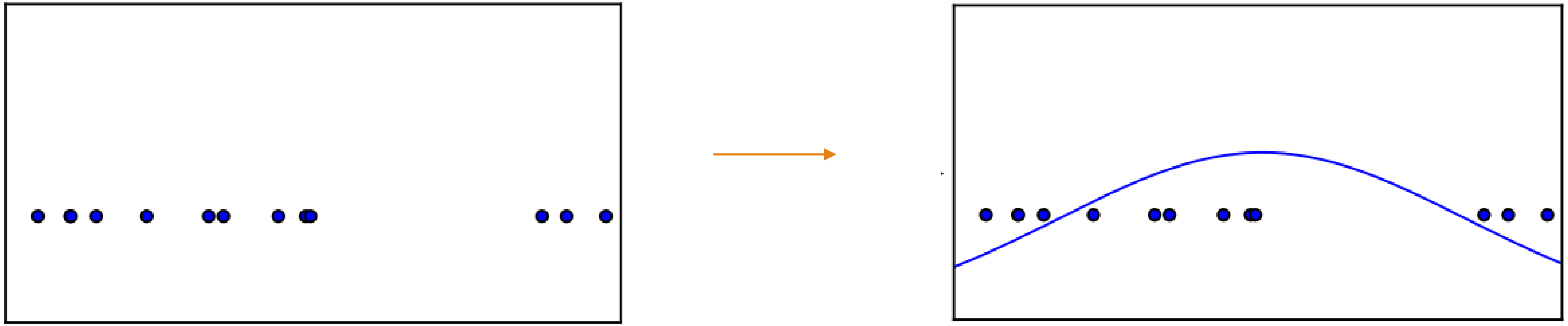
Problems

- Design complex enough model that meets data complexity
- At the same time, make sure model is computationally tractable
- More details in the next lecture



Generative modeling: Case I

- Density estimation

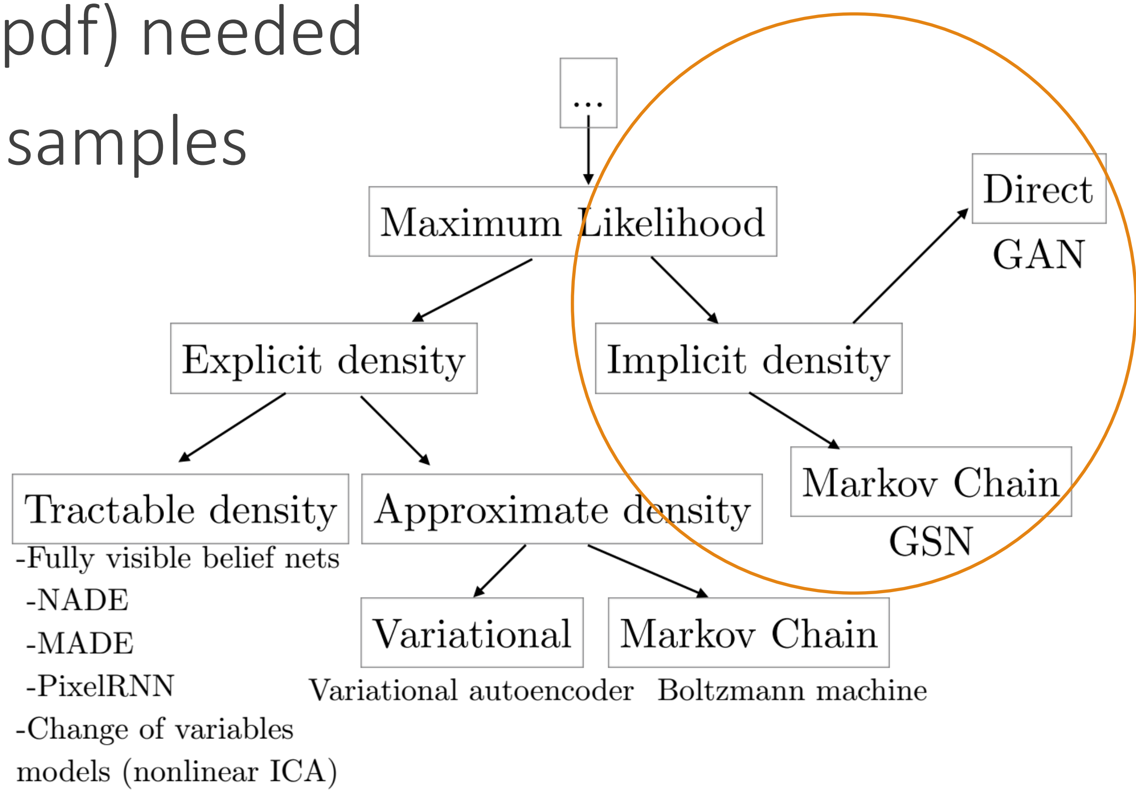


Train set

Fitted model

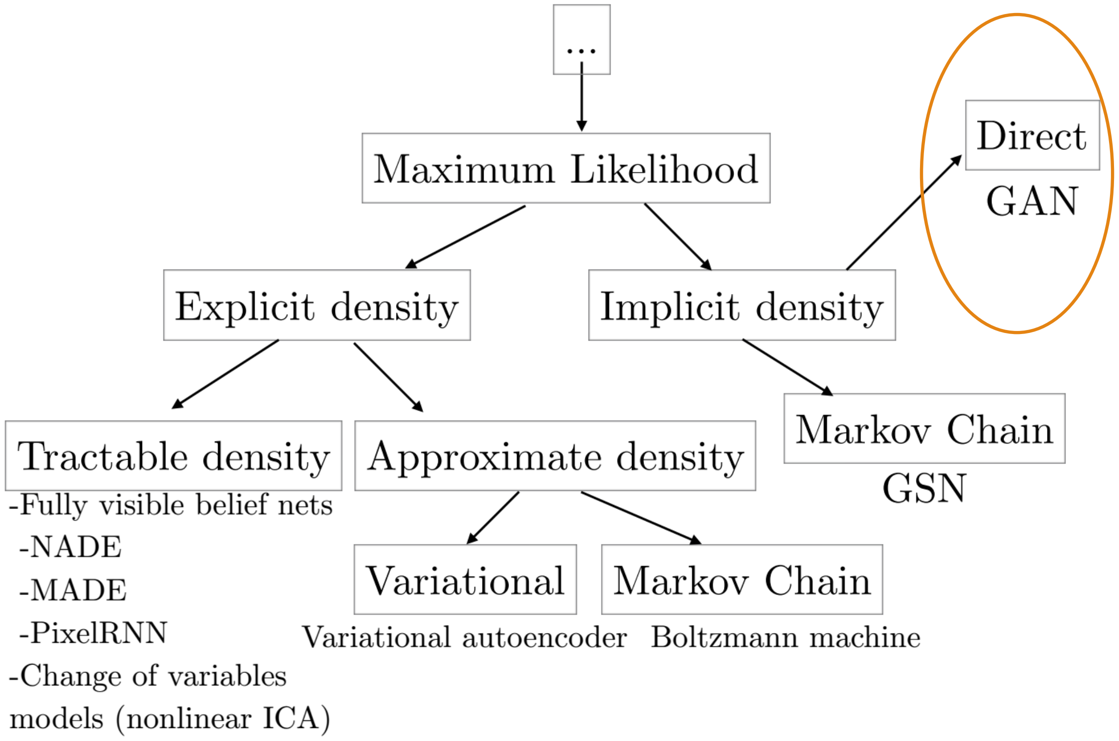
Implicit density models

- No explicit probability density function (pdf) needed
- Instead, a sampling mechanism to draw samples from the pdf without knowing the pdf



Implicit density models: GANs

- Sample data in parallel
- Few restrictions on generator model
- No Markov Chains needed
- No variational bounds
- Better qualitative examples
  - Weak but true



Generative modeling: Case II

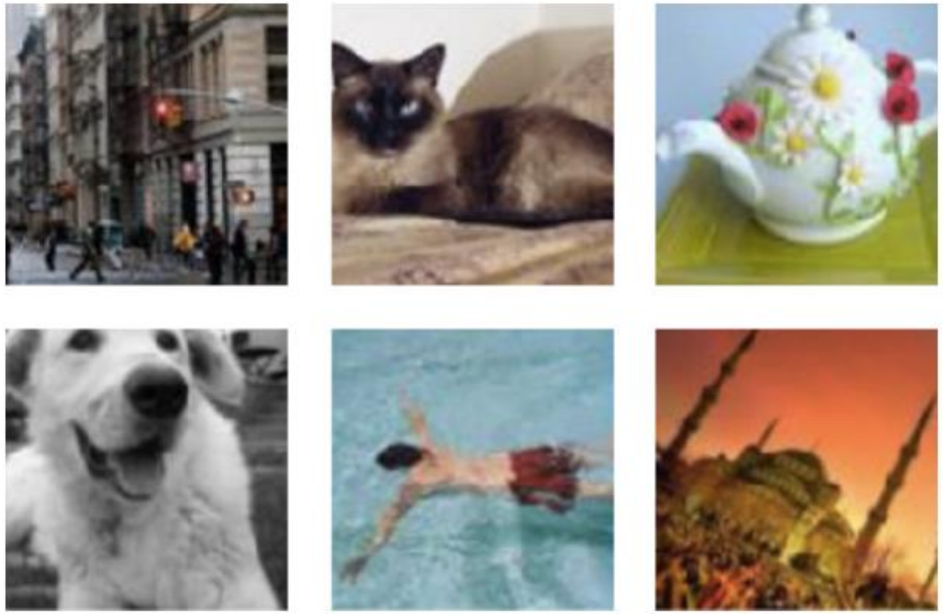
- Sample Generation

Train examples

Generative modeling: Case II

- Sample Generation

Train examples



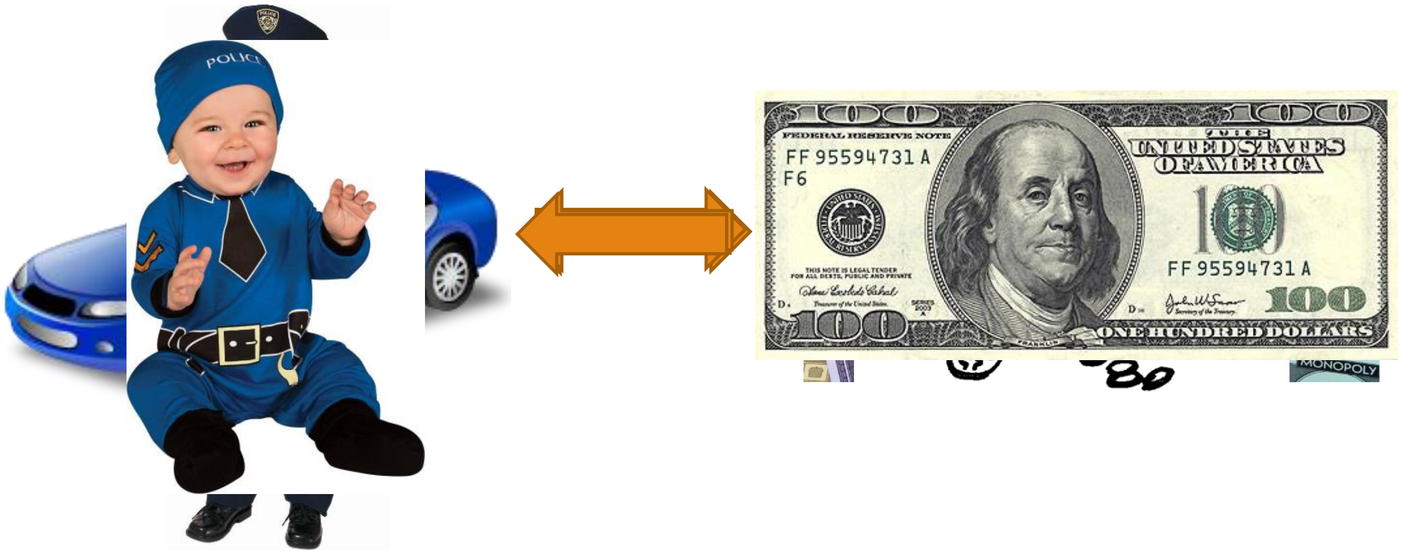
New samples (ideally)

What is a GAN?

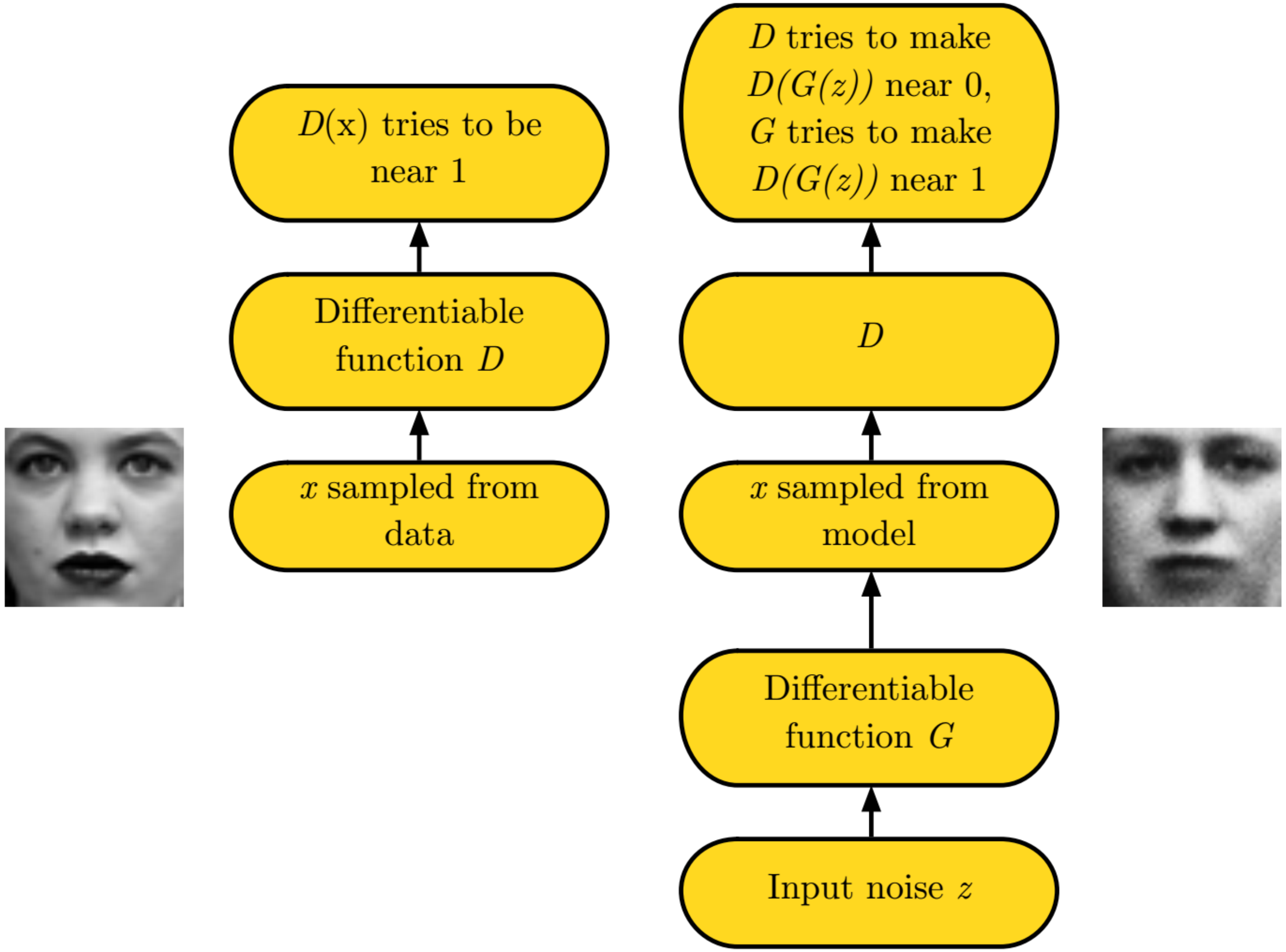
- **G**enerative
  - You can sample novel input samples
  - E.g., you can literally “create” images that never existed
- **A**dversarial
  - Our generative model G learns adversarially, by fooling an discriminative oracle model D
- **N**etwork
  - Implemented typically as a (deep) neural network
  - Easy to incorporate new modules
  - Easy to learn via backpropagation

GAN: Intuition

- Assume you have two parties
  - Police: wants to recognize fake money as reliably as possible
  - Counterfeiter: wants to make as realistic fake money as possible
- The police forces the counterfeiter to get better (and vice versa)
- Solution relates to Nash equilibrium

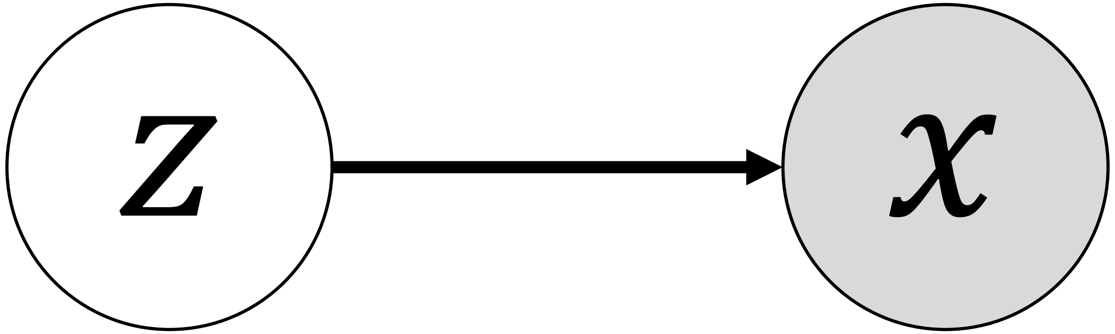


GAN: Pipeline



Generator network $x = G(z; \theta^{(\mathrm{G})})$

- Must be differentiable
- No invertibility requirement
- Trainable for any size of z
- Can make conditionally Gussian given z, but no strict requirement



Generator & Discriminator: Implementation

- The discriminator is just a standard neural network
- The generator looks like an inverse discriminator

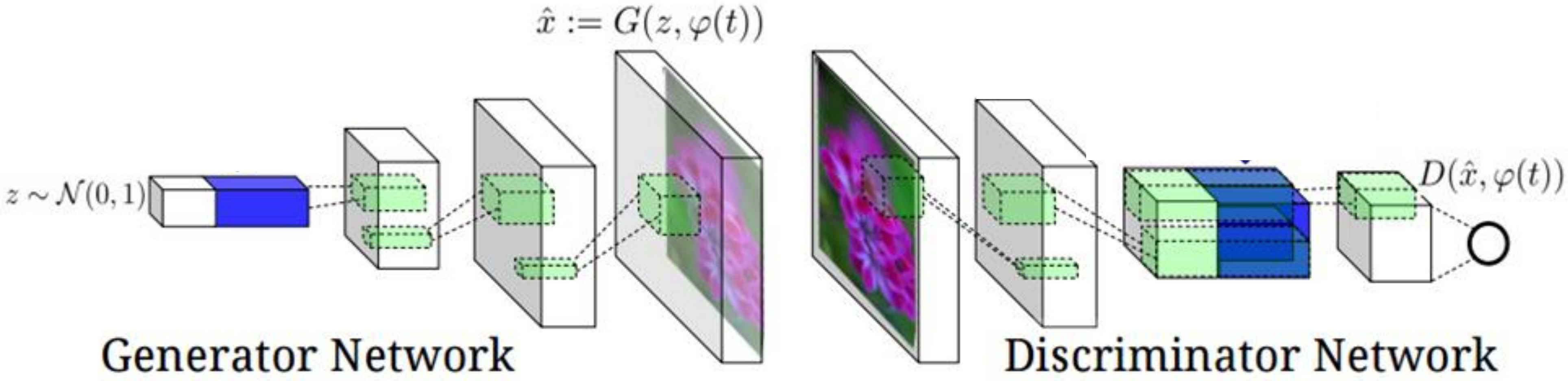


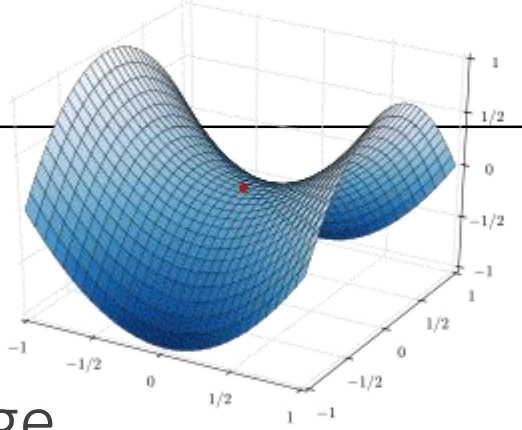
Figure 2. Our text-conditional convolutional GAN architecture. Text encoding $\varphi(t)$ is used by both generator and discriminator. It is projected to a lower-dimensions and depth concatenated with image feature maps for further stages of convolutional processing.

Network Architecture

Training definitions

- Minimax
- Maximin
- Heuristic, non-saturating game
- Max likelihood game

Minimax Game



- $J^{(D)} = -\frac{1}{2}\mathbb{E}_{x\sim p_{data}} \log D(x) \; - \frac{1}{2}\mathbb{E}_{z\sim p_z} \log(1 - D(G(z)))$
- $D(x) = 1 \rightarrow$ The discriminator believes that x is a true image
- $D(G(z)) = 1 \rightarrow$ The discriminator believes that $G(z)$ is a true image

- Equilibrium is a saddle point of the discriminator loss
- Resembles Jensen-Shannon divergence
- Generator minimizes the log-probability of the discriminator being correct

Minimax Game

- For the simple case of zero-sum game

$$J^{(G)} = -J^{(D)}$$

- So, we can summarize game by

$$V\left(\theta^{(\mathrm{D})}, \theta^{(\mathrm{G})}\right) = -J^{(D)}(\theta^{(\mathrm{D})}, \theta^{(\mathrm{G})})$$

- Easier theoretical analysis

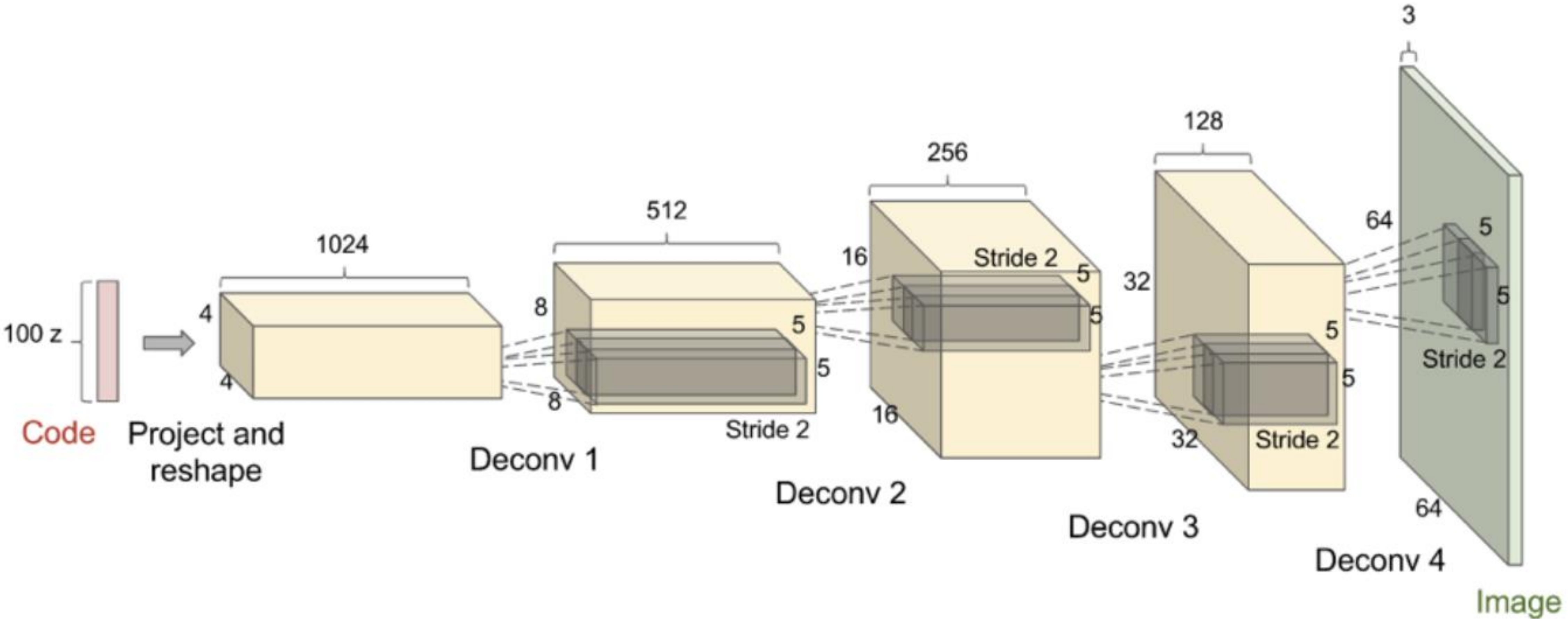
- In practice not used → when the discriminator starts to recognize fake samples, the generator gradients vanish

Heuristic non-saturating game

- $J^{(D)} = -\frac{1}{2}\mathbb{E}_{x\sim p_{data}} \log D(x) \; - \frac{1}{2}\mathbb{E}_{z\sim p_z} \log(1 - D(G(z))$
- $J^{(G)} = -\frac{1}{2}\mathbb{E}_{z\sim p_z} \log(D(G(z))$

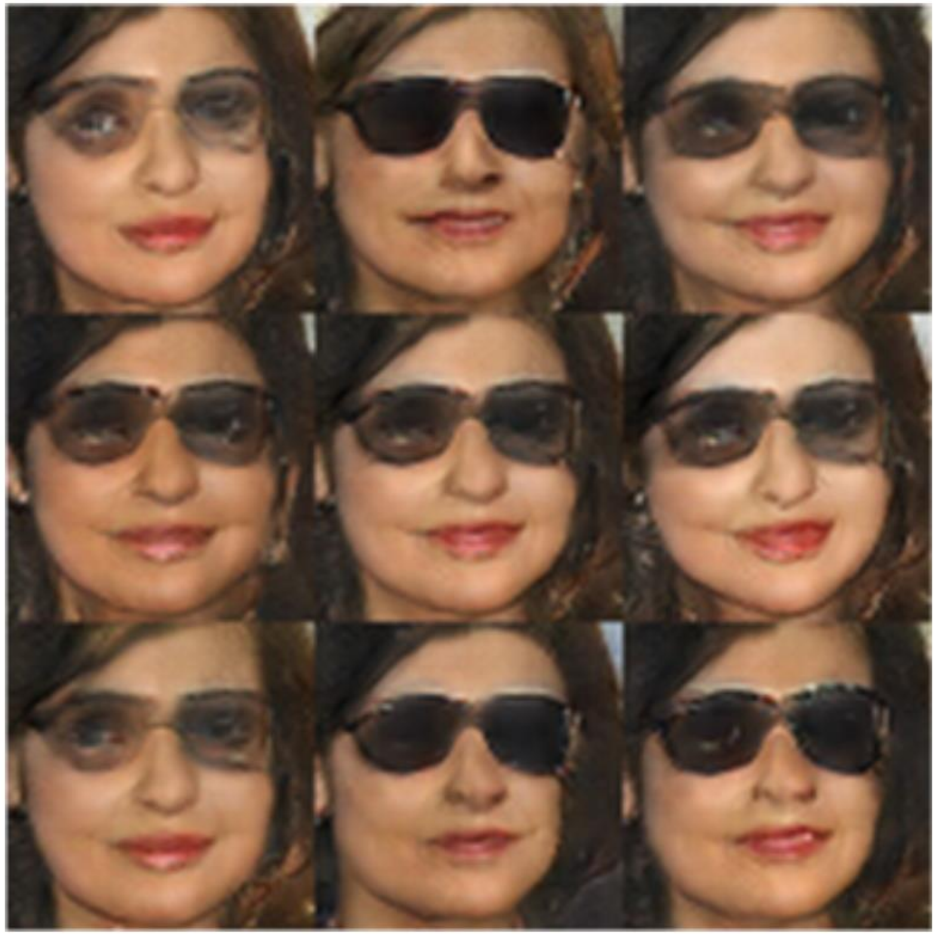
- Equilibrium not any more describable by single loss
- Generator maximizes the log-probability of the discriminator being mistaken
  - Good $G(z)$ → $\mathrm{D}\big(G(z)\big) = 1$ → $J^{(G)}$ is maximized
- Heuristically motivated; generator can still learn even when discriminator successfully rejects all generator samples

DCGAN Architecture



Examples

Even vector space arithmetics …

 -  +  = 

Man with glasses

Man

Woman

Woman with glasses

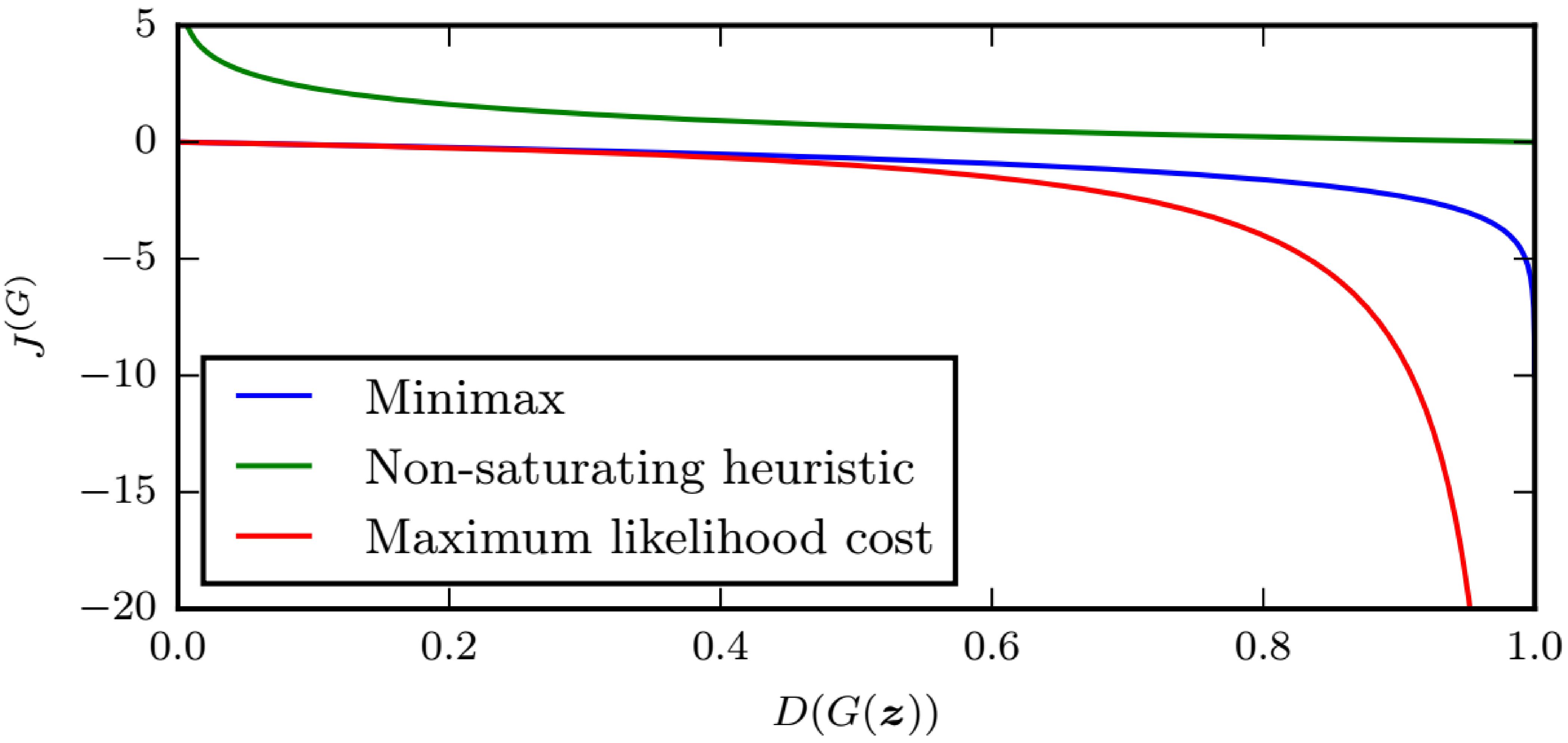
Modifying GANs for Max-Likelihood

- $J^{(D)} = -\frac{1}{2}\mathbb{E}_{x\sim p_{data}} \log D(x) \; - \frac{1}{2}\mathbb{E}_z \log(1 - D(G(z)))$
- $J^{(G)} = -\frac{1}{2}\mathbb{E}_z \log(\sigma^{-1}(D\big(G(z)\big))$

- When discriminator is optimal, the generator gradient matches that of maximum likelihood

- "On distinguishability Criteria for Estimating Generative Models", Goodfellow 2014

Comparison of Generator Losses

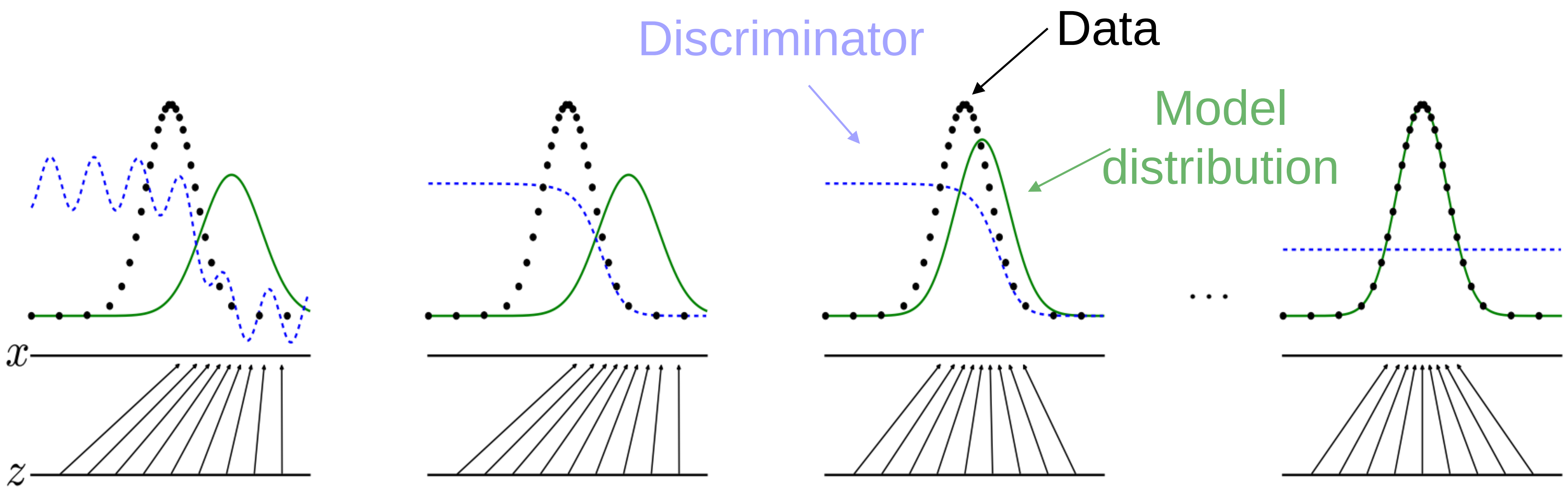


Optimal discriminator

- Optimal $D(x)$ for any $p_{data}(x)$ and $p_{model}(x)$ is always

$$D(x) = \frac{p_{data}(x)}{p_{data}(x) + p_{model}(x)}$$

- Estimating this ratio with supervised learning (discriminator) is the key



Why is this the optimal discriminator?

- $L(D,G) = \int_x p_r(x) \log D(x) + p_g(x) \log\big(1 - D(x)\big)\, dx$
  - Minimize $L(D,G)$ w.r.t. D
- By setting $\tilde{x} = D(x), A = p_r(x), B = p_g(x)$ and setting $\frac{dL}{d\tilde{x}} = 0$ and ignoring the integral because we sample over all x

$$D^*(x) = \frac{p_r(x)}{p_r(x) + p_g(x)}$$

- For an **optimal** generator: $p_g(x) \to p_r(x)$ we have

$$D^*(x) = \frac{1}{2}$$
$$L(G^*, D^*) = -2 \log 2$$

GANs and Jensen-Shannon divergence

- By expanding the Jensen-Shannon divergence, we have

$$D_{JS}(p_r||p_g) = \frac{1}{2} D_{KL}(p_r||\frac{p_r + p_g}{2}) + \frac{1}{2} D_{KL}(p_g||\frac{p_r + p_g}{2})$$

$$= \frac{1}{2}\left(\log 2 + \int_x p_r(x) \log \frac{p_r(x)}{p_r(x) + p_g(x)} dx + \log 2\right.$$

GANs and Jensen-Shannon divergence

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https://lilianweng.github.io/lil-log/2017/08/20/from-GAN-to-WGAN.html

Is the divergence important?

- Does the divergence make a difference?
- Is there a difference between KL-divergence, Jensen-Shannon divergence, …

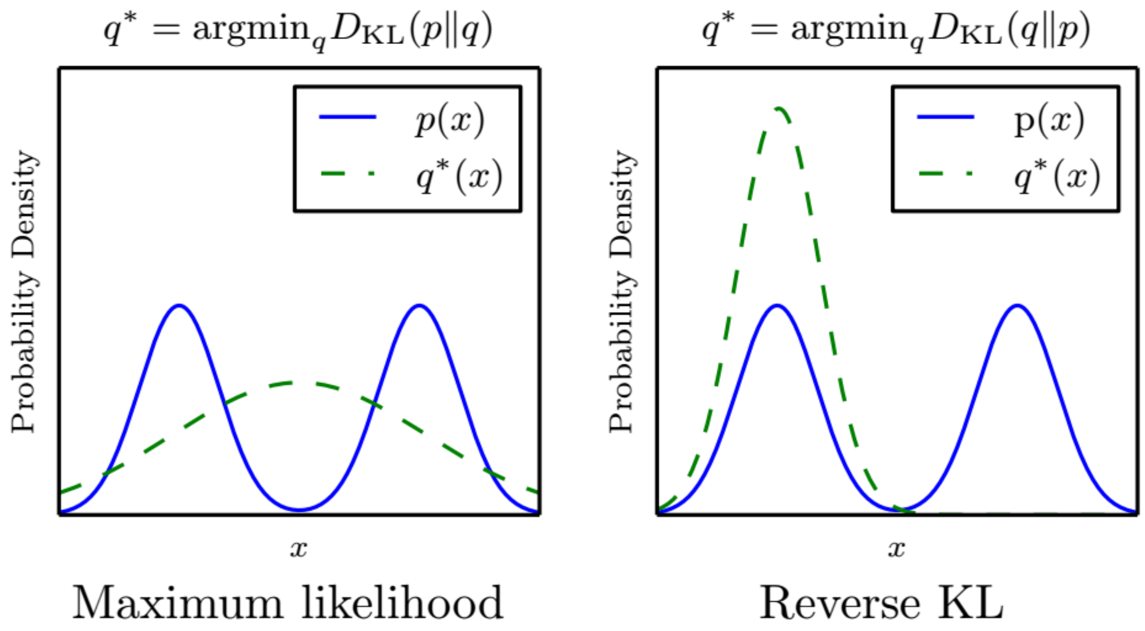
$$D_{KL}(p_r||p_g) = \int_x p_r \log \frac{p_r}{p_g} dx$$

$$D_{JS}(p_r||p_g) = \frac{1}{2} D_{KL}(p_r||\frac{p_r + p_g}{2}) + \frac{1}{2} D_{KL}(p_g||\frac{p_r + p_g}{2})$$

- Let’s check the KL-divergence

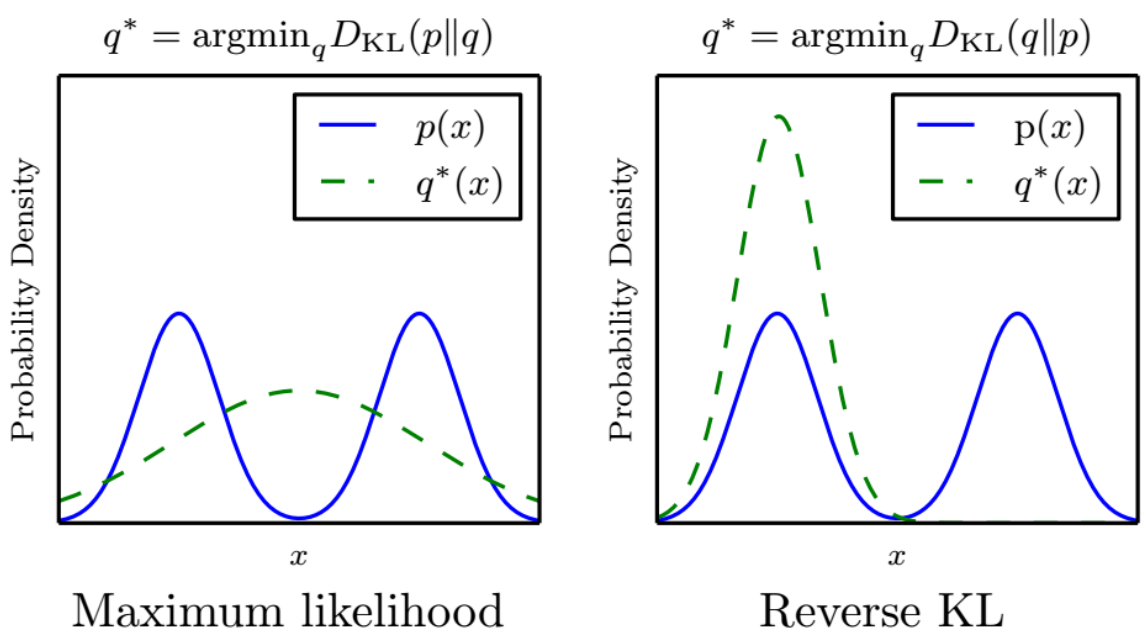
Is the divergence important?

- Forward KL divergence: $D_{KL}(p(x)||q^*(x))$ → high probability everywhere that the data occurs
- Backward KL divergence: $D_{KL}(q^*(x)||p(x))$ → low probability wherever the data does not occur
- Which version makes the model “c



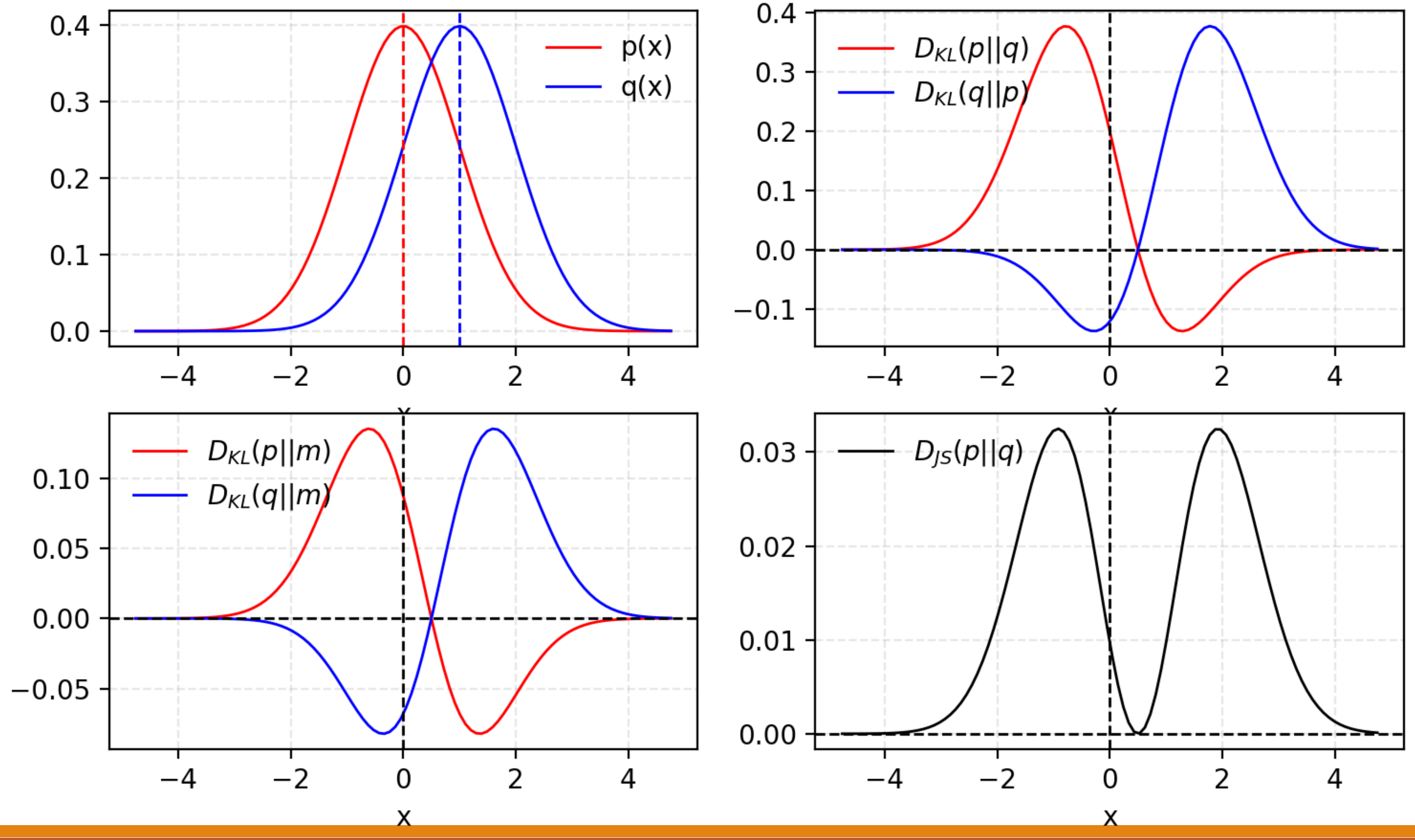
Is the divergence important?

- $D_{KL}(p(x)||q^*(x))$→ high probability everywhere that the data occurs
- $D_{KL}(q^*(x)||p(x))$ → low probability wherever the data does not occur
- Which version makes the model "conservative"?
- $D_{KL}(q^*(x)||p(x)) = \int q^*(x) \log \frac{q^*(x)}{p(x)}$
  - Avoid areas where $p(x) \to 0$
- Zero-forcing
  - $q^*(x) \to 0$ in areas when approximation $\frac{q^*(x)}{p(x)}$ cannot be good



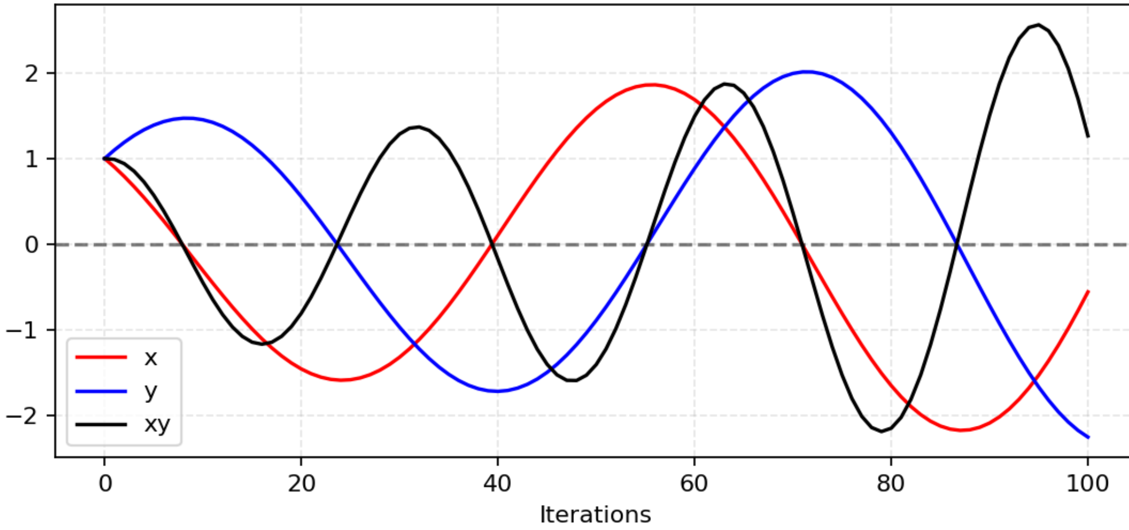
KL vs JS

- JS is symmetric, KL is not



GAN Problems: Reaching Nash equilibrium causes instabilities

- GANs is a mini-max optimization
  - Non-cooperative game with a tied objective
- Training is not always easy
  → When optimizing one player/network, we might hurt the other one
  → oscillations
- E.g., assume we have two players $f(x) = xy$ one step at a time
  - Player 1 minimizes: $\min_{x} f_1(x) = xy \Rightarrow \frac{df_1}{dx} = y$
    $\Rightarrow x_{t+1} = x_t - \eta \cdot y$
  - Player 2 minimizes: $\min_{y} f_2(x) = -xy \Rightarrow \frac{df_2}{dx} = -x$
    $\Rightarrow y_{t+1} = y_t + \eta \cdot x$



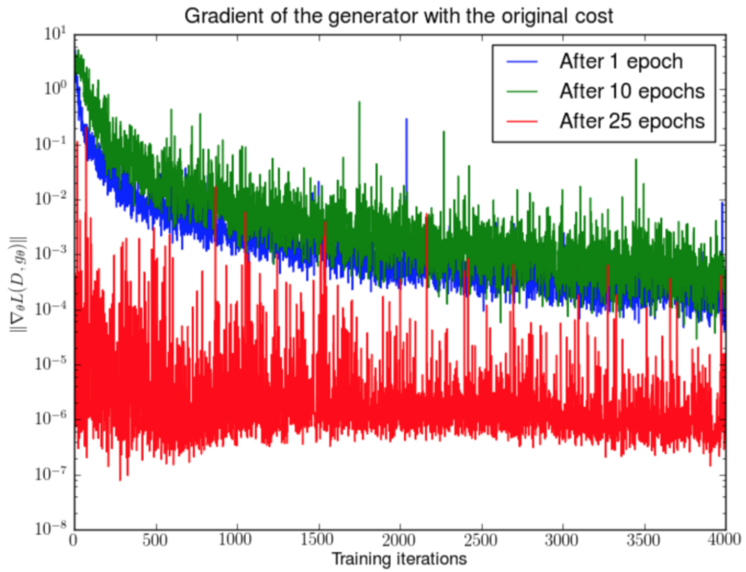
https://lilianweng.github.io/lil-log/2017/08/20/from-GAN-to-WGAN.html

GAN Problems: Vanishing Gradients

$$J^{(D)} = -\frac{1}{2}\mathbb{E}_{x \sim p_{data}} \log D(x) \; - \frac{1}{2}\mathbb{E}_z \log(1 - D(G(z)))$$

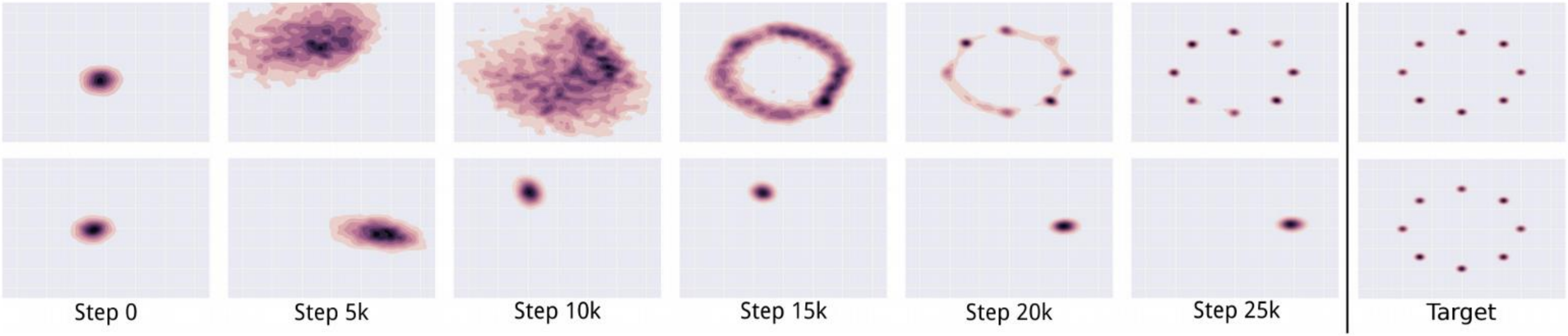
$$J^{(G)} = -\frac{1}{2}\mathbb{E}_z \log(D(G(z)))$$

- If the discriminator is quite bad, then the generator does not get reasonable gradients
- But, if the discriminator is perfect, $D(x) = D^*(x)$, the gradients go to 0
  - No learning anymore
- Bad when this happens early in the training
  - Easier to train the discriminator than the generator



GAN Problems: Mode collapse

- Very low variability
- It is safer for the generator to produce samples from the mode it knows it approximates well



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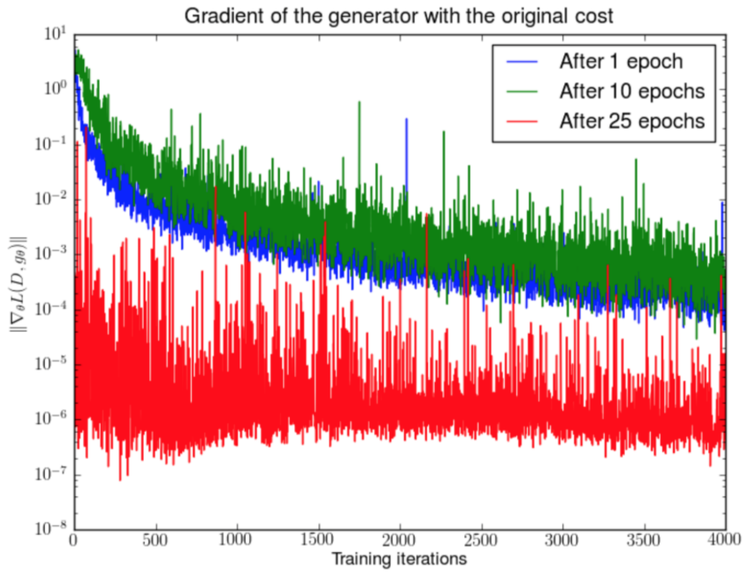
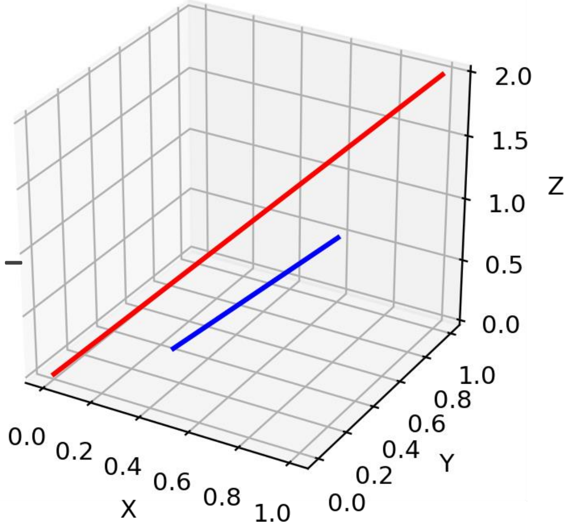
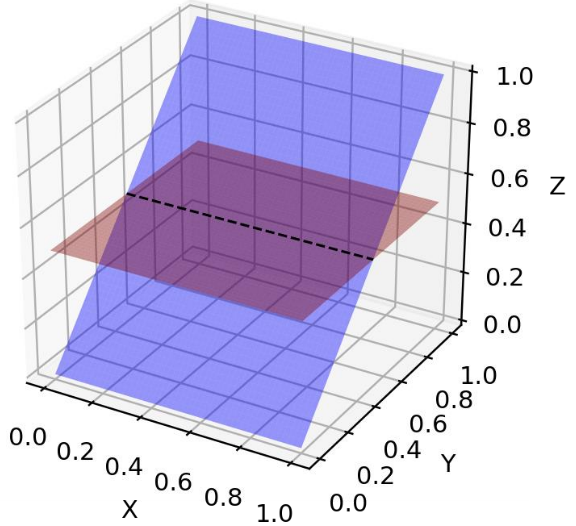


GAN Problems: Low dimensional supports

- Data lie in low-dim manifolds
- However, the manifold is not known
- During training p_g is not perfect either, especially in the start
- So, the support of p_r and p_g is non-overlapping and disjoint
  → not good for KL/JS divergences



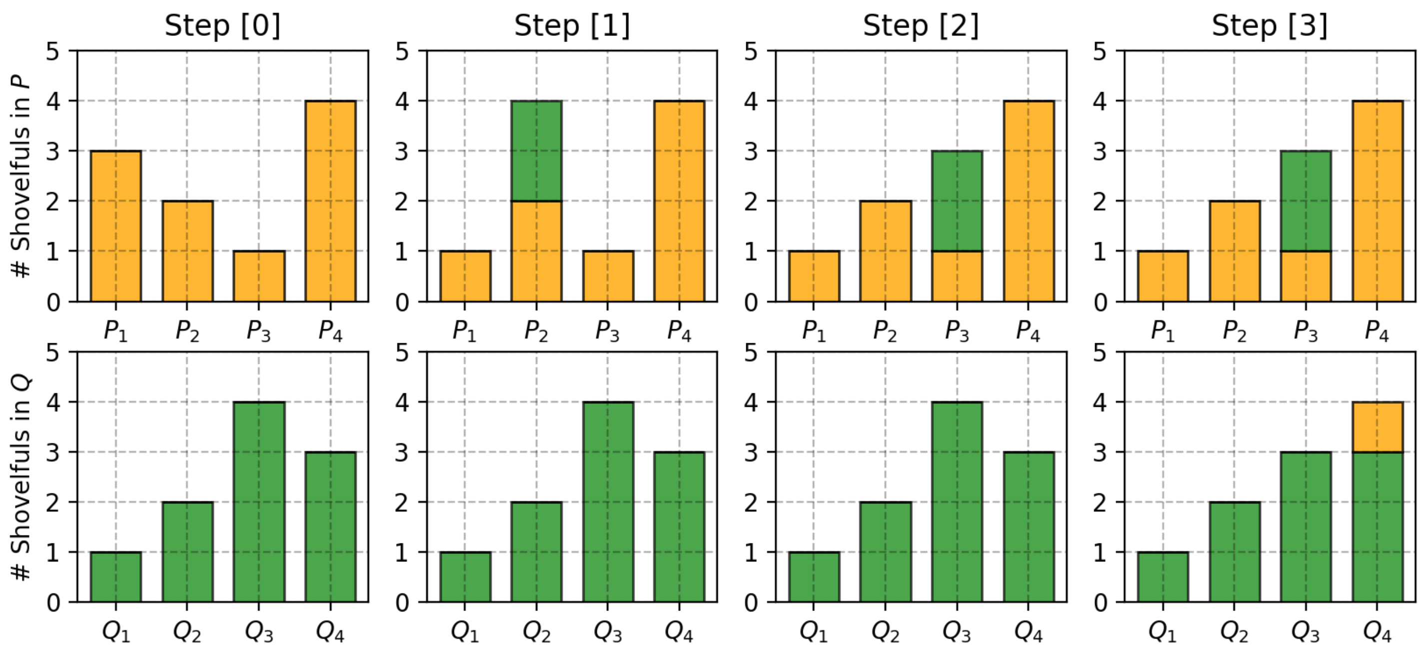


Wasserstein GAN

- Instead of KL/JS, use Wasserstein (Earth Mover's) Distance

$$W(p_r, p_g) = \inf_{\gamma \sim \Pi(\mathrm{p_r}, \mathrm{p_g})} \mathrm{E}_{(\mathrm{x,y}) \sim \gamma} |x - y|$$

- Even for non-overlapping supports, the distance is meaningful



Feature matching

- Instead of matching image statistics, match feature statistics

$$J^{(D)} = \left\| \mathbb{E}_{x\sim p_r} f(x) - \mathbb{E}_{z\sim p_z} f\big(G(z)\big) \right\|_2^2$$

- f can be any statistic of the data, like the mean or the median

Training procedure

- Use SGD-like algorithm of choice
  - Adam Optimizer is a good choice
- Use two mini-batches simultaneously
  - The first mini-batch contains real examples from the training set
  - The second mini-batch contains fake generated examples from the generator
- Optional: run k-steps of one player (e.g. discriminator) for every step of the other player (e.g. generator)

Use labels if possible

- Learning a conditional model $p(y|x)$ is often generates better samples
  - Denton et al., 2015
- Even learning $p(x, y)$ makes samples look more realistic
  - Salimans et al., 2016
- Conditional GANs are a great addition for learning with labels

One-sided label smoothing

- Default discriminator cost:

```
cross_entropy(1., discriminator(data))
+ cross_entropy(0., discriminator(samples))
```

- One-sided label smoothing:

```
cross_entropy(0.9, discriminator(data))
+ cross_entropy(0., discriminator(samples))
```

- Do not smooth negative labels:

```
cross_entropy(1.-alpha, discriminator(data))
+ cross_entropy(beta, discriminator(samples))
```

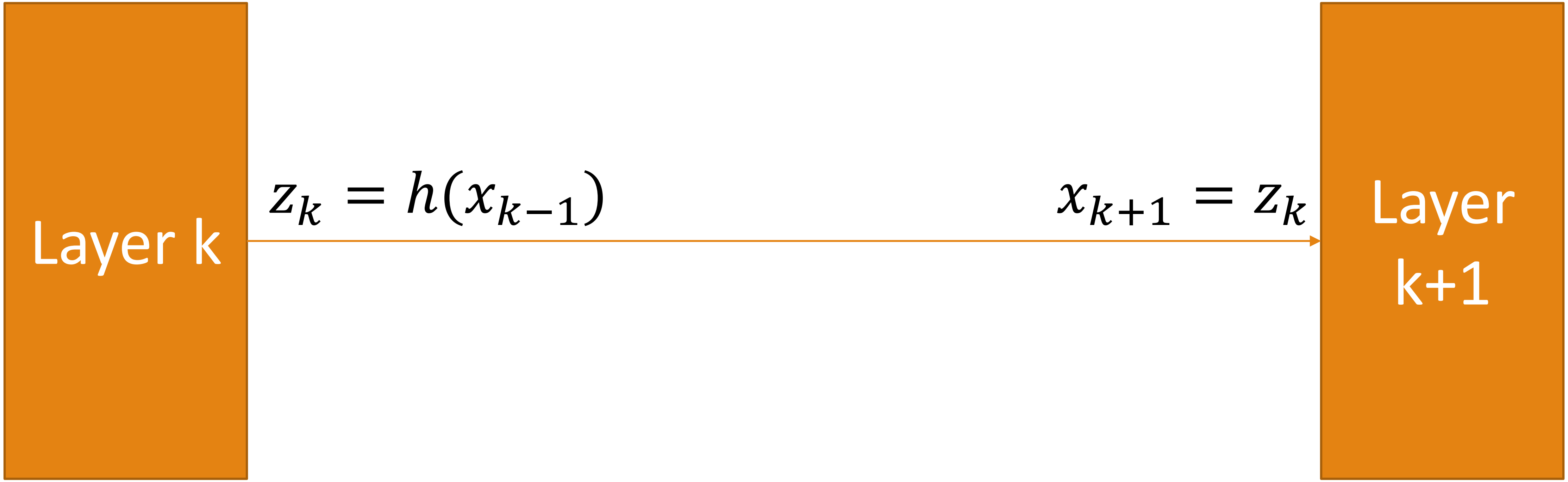
Benefits of label smoothing

- Max likelihood often is overconfident
  - Might return accurate prediction, but too high probabilities
- Good regularizer
  - Szegedy et al., 2015
- Does not reduce classification accuracy, only confidence
- Specifically for GANs
  - Prevents discriminator from giving very large gradient signals to generator
  - Prevents extrapolating to encourage extreme samples

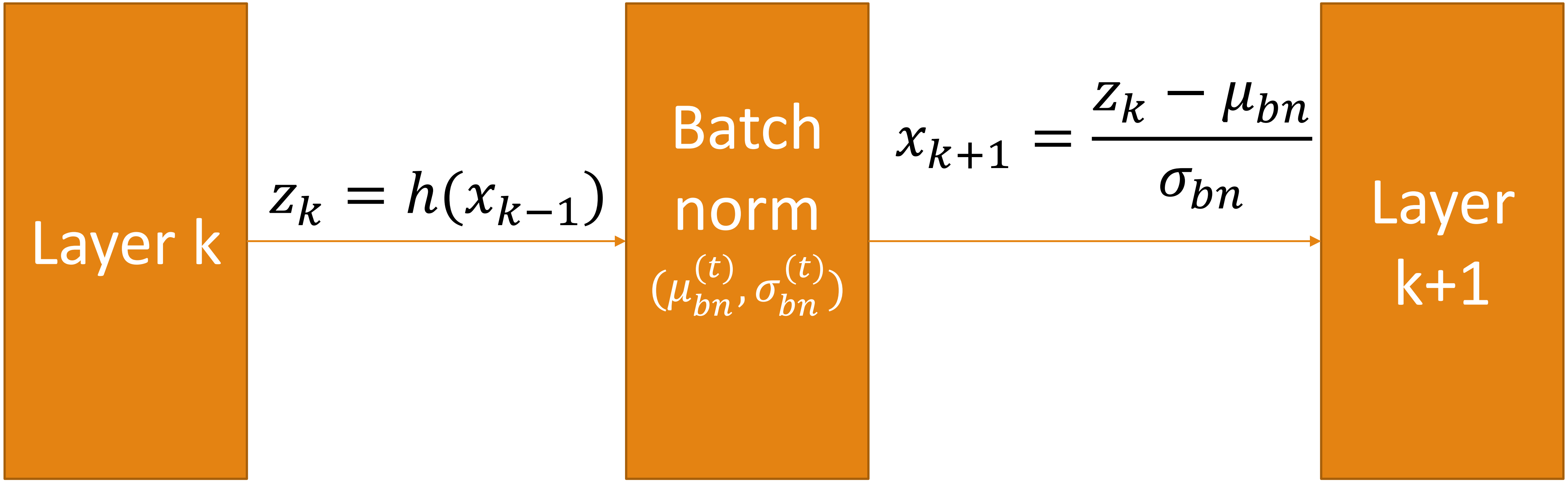
Batch normalization

- Generally, good practice for neural networks
- Given inputs $X = \{x^{(1)}, x^{(2)}, \ldots, x^{(m)}\}$
- Compute mean and standard deviation of features of X: μ_{bn}, σ_{bn}
- Normalize features
  - Subtract mean, divide by standard deviation

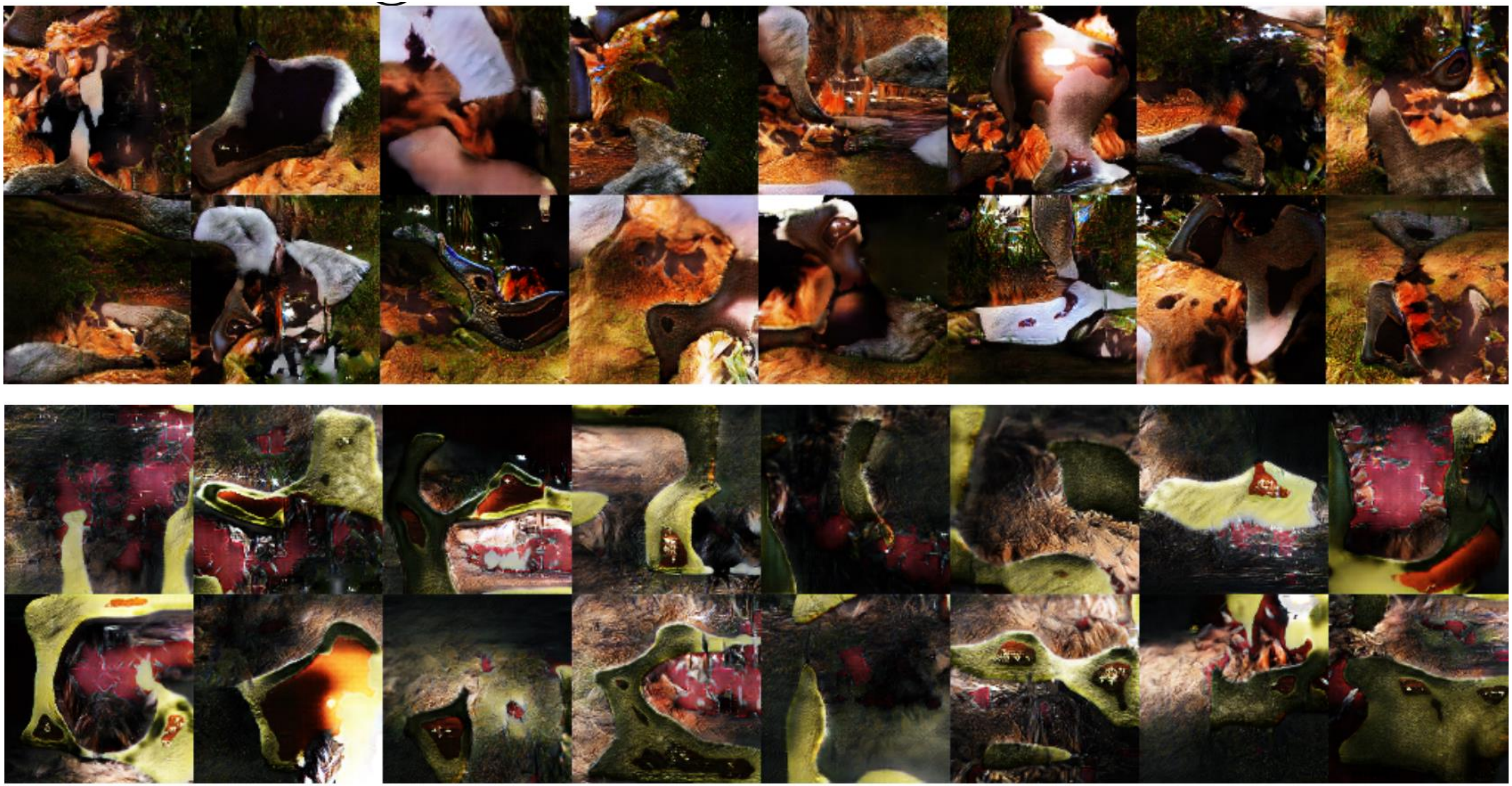
Batch normalization: Graphically



Batch normalization: Graphically



But, can cause strong intra-batch correlation



Reference batch normalization

- Training with two mini-batches
- One fixed reference mini-batch for computing mean and standard deviation
- The other for doing the training as usual
- Proceed as normal, only use the mean and standard deviation for the batch norm from the fixed reference mini-batch
- Problem: Overfitting to the reference mini-batch

	Standard mini-batch	Reference mini-batch
Iteration 1	$\frac{dJ^{(1)}}{d\theta}$	μ_{bn}, σ_{bn}
Iteration 2	$\frac{dJ^{(2)}}{d\theta}$	μ_{bn}, σ_{bn}
Iteration 3	$\frac{dJ^{(3)}}{d\theta}$	μ_{bn}, σ_{bn}

Solution: Virtual batch normalization

- Mini-batch= standard mini-batch + reference, fixed mini-batch

	Standard mini-batch	Reference mini-batch
Iteration 1	$\frac{dJ^{(1)}}{d\theta}$	$\mu_{bn}^{(R)}, \sigma_{bn}^{(R)}$
Iteration 2	$\frac{dJ^{(2)}}{d\theta}$	$\mu_{bn}^{(R)}, \sigma_{bn}^{(R)}$
Iteration 3	$\frac{dJ^{(3)}}{d\theta}$	$\mu_{bn}^{(R)}, \sigma_{bn}^{(R)}$

Balancing Generator & Discriminator

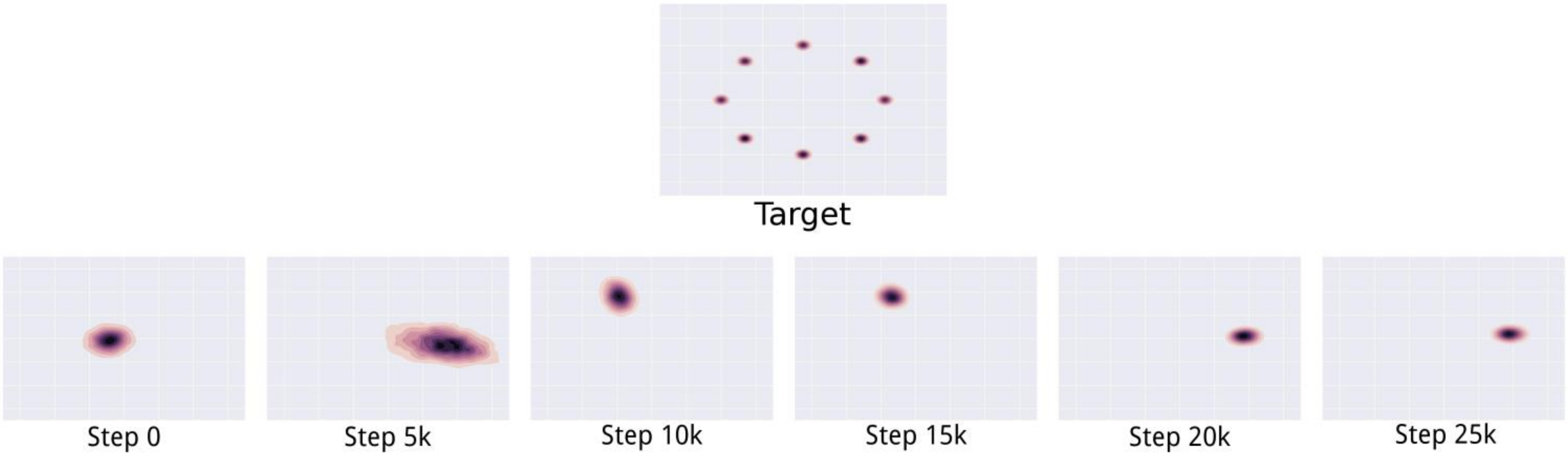
- Usually the discriminator wins
  - That's good, in that the theoretical justification assume a perfect discriminator
- Usually the discriminator network is bigger than the generator
- Sometimes running discriminator more often than generator works better
  - However, no real consensus
- Do not limit the discriminator to avoid making it too smart
  - Better use non-saturating cost
  - Better use label smoothing

Open Question: Non-convergence

- Optimization is tricky and unstable
  - finding a saddle point does not imply a global minimum
- An equilibrium might not even be reached
- Mode-collapse is the most severe form of non-convergence

Open Question: Mode collapse

- Discriminator converges to the correct distribution
- Generator however places all mass in the most likely point



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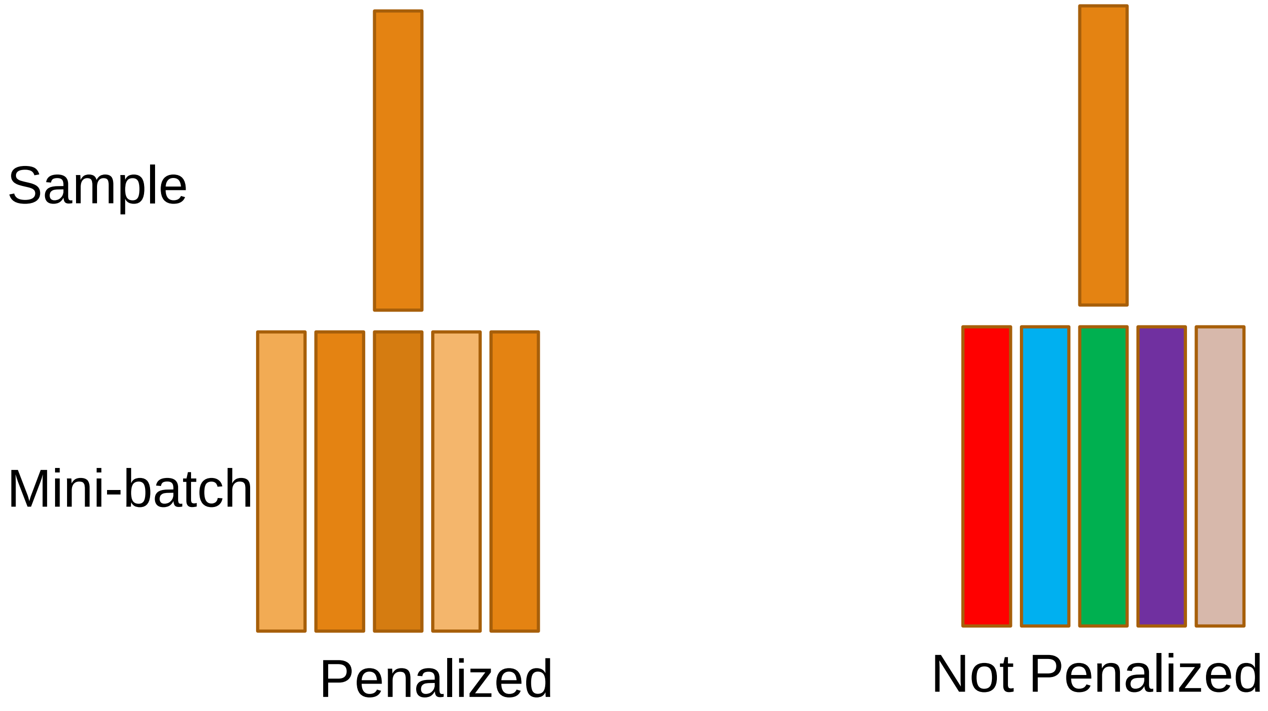
- Discriminator converges to the correct distribution
- Generator however places all mass in the most likely point
- Problem: low sample diversity

Minibatch features

- Classify each sample by comparing to other examples in the mini-batch
- If samples are too similar, the model is penalized



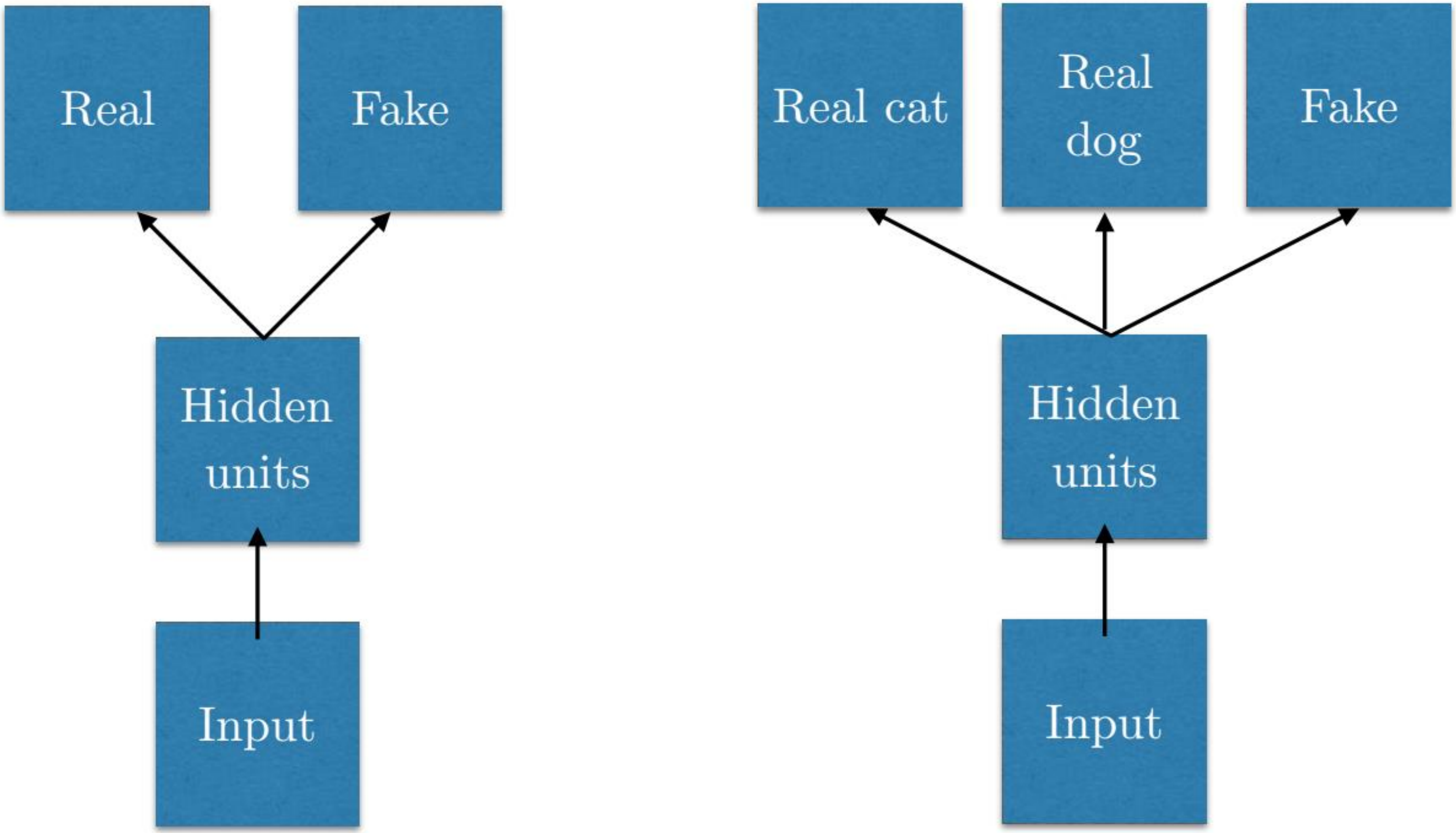
Open Question: Evaluation of GANs

- Despite the nice images, who cares?
- It would be nice to quantitatively evaluate the model
- For GANs it is even hard to estimate the likelihood

Open Question: Discrete outputs

- The generator must be differentiable
- It cannot be differentiable if outputs are discrete
- E.g., harder to make it work for text
- Possible workarounds
  - REINFORCE [Williams, 1992]
  - Concrete distribution [Maddison et al., 2016]
  - Gumbel softmax [Jang et al., 2016]
  - Train GAN to generate continuous embeddings

Open Question: Semi-supervised classification



Interpretable latent codes

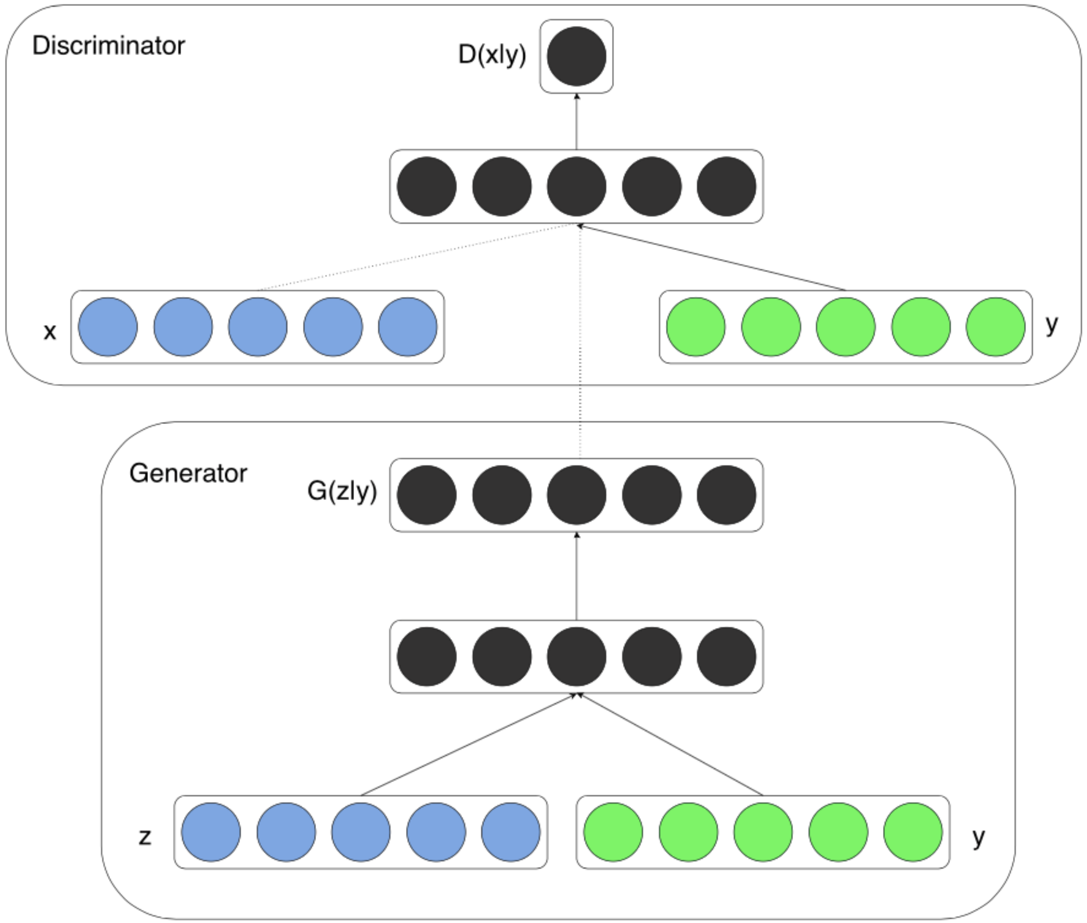
- InfoGAN [Chen et al., 2016]

(a) Azimuth (pose) (b) Elevation

(c) Lighting (d) Wide or Narrow

GAN spinoffs

- Conditional GANs
  - Standard GANs have **no encoder**!
- Actor-Critic
  - Related to Reinforcement Learning



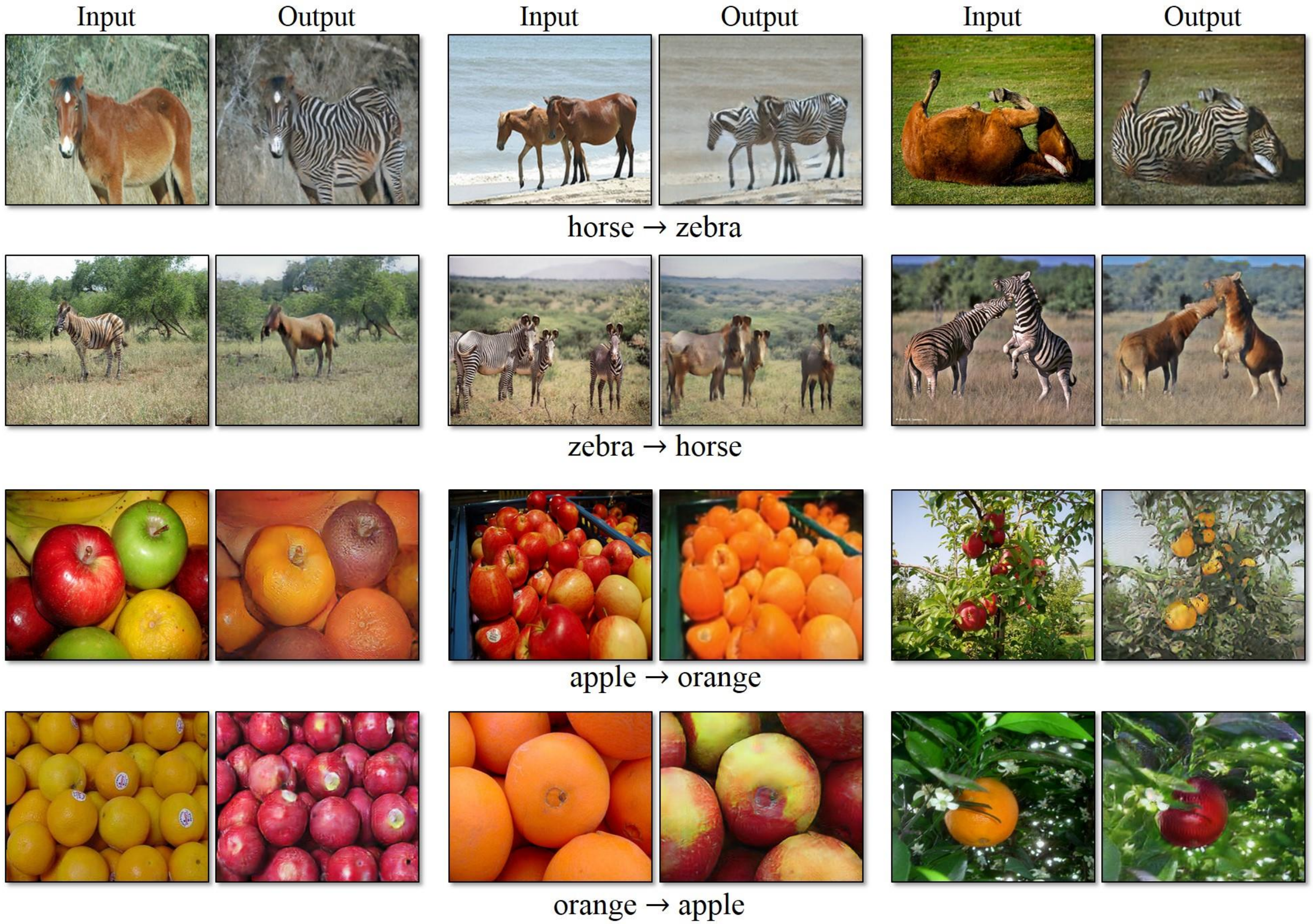
Conditional GAN

Connections to Reinforcement Learning

- GANs interpreted as actor-critic [Pfau and Vinyals, 2016]
- GANs as inverse reinforcement learning [Finn et al., 2016]
- GANs for imitation learning [Ho and Ermin 2016]

Application: Image to Image translation

Application: Style transfer



Application: Face generation

- https://www.youtube.com/watch?v=XOxxPcy5Gr4

Summary

- GANs are generative models using supervised learning to approximate an intractable cost function
- GANs can simulate many cost functions, including max likelihood
- Finding Nash equilibria in high-dimensional, continuous, non-convex games is an important open research problem
- GAN research is in its infancy, most works published only in 2016. Not mature enough yet, but very compelling results