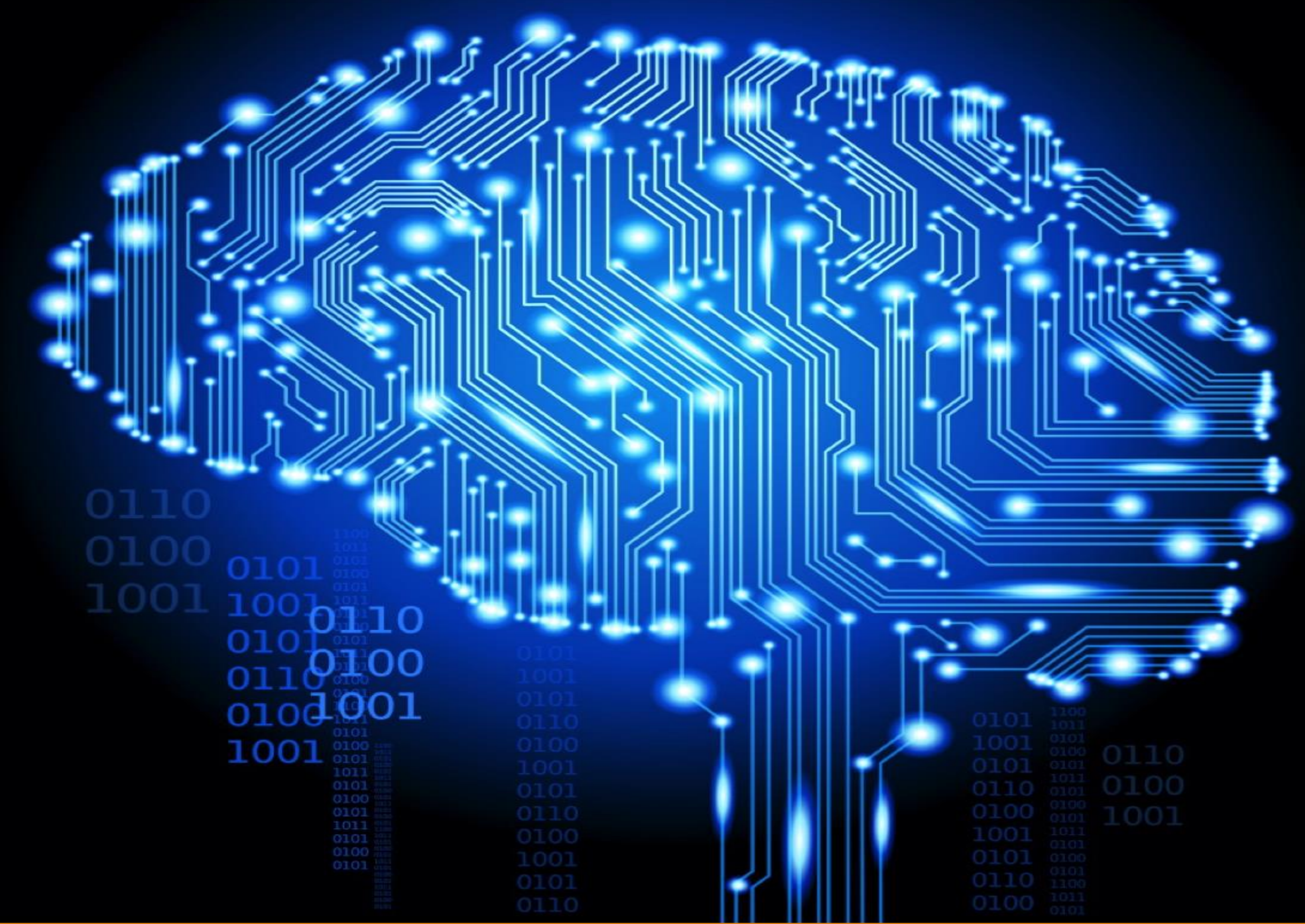


Lecture 7: Generative Adversarial Networks

Efstratios Gavves

11 suspected cases for plagiarism

- The students have already been notified
- The cases have been forwarded to the examinations board, who are responsible for evaluating the case
- Plagiarizing is bad
  - for all other students, who did the work
  - for the degree, which loses its values
  - For the student’s money, they actually lose the opportunity to learn
  - For the teaching stuff, who put all the effort in delivering the course as well as they can
- In short, please **don’t do it** otherwise there are consequences.

Lecture overview

- Gentle intro to generative models
- Generative Adversarial Networks
- Variants of Generative Adversarial Networks

Generative models

(a) EBGAN (64x64)

(b) Our results (128x128)

Types of Learning

- Generative modelling
  - Learn the joint pdf: $p(x, y)$
  - Model the world → Perform tasks, *e.g.* use Bayes rule to classify: $p(y|x)$
  - Naïve Bayes, Variational Autoencoders, GANs
- Discriminative modelling
  - Learn the conditional pdf: $p(y|x)$
  - Task-oriented
  - E.g., Logistic Regression, SVM

Types of Learning

- What to pick?
  - V. Vapnik: *"One should solve the [classification] problem directly and never solve a more general [and harder] problem as an intermediate step."*
- Typically, discriminative models are selected to do **the** job
- Generative models give us more theoretical guarantees that the model is going to work as intended
  - Better generalization
  - Less overfitting
  - Better modelling of causal relationships

Applications of generative modeling?

Applications of generative modeling?

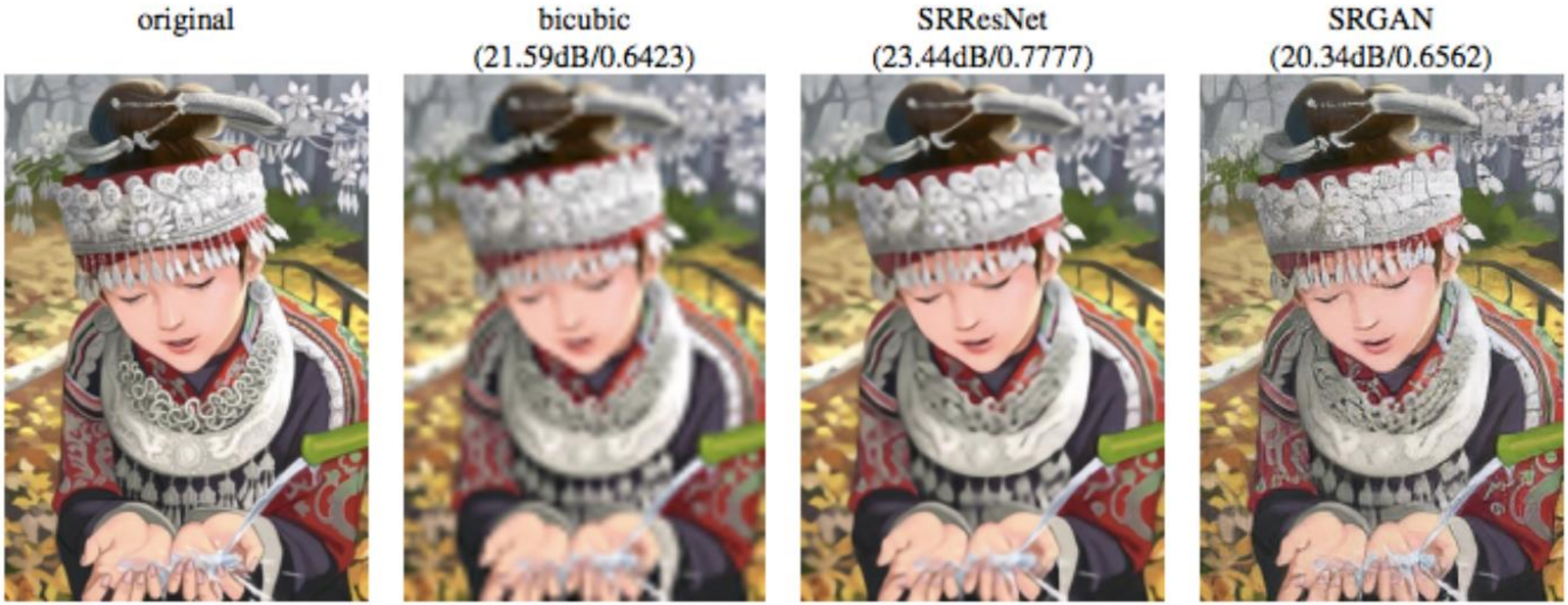
- Act as a regularizer in discriminative learning
  - Discriminative learning often too goal-oriented
  - Overfitting to the observations
- Semi-supervised learning
  - Missing data
- Simulating “possible futures” for Reinforcement Learning
- Data-driven generation/sampling/simulation

Applications: Image Generation

(a) Generated by LSGANs.

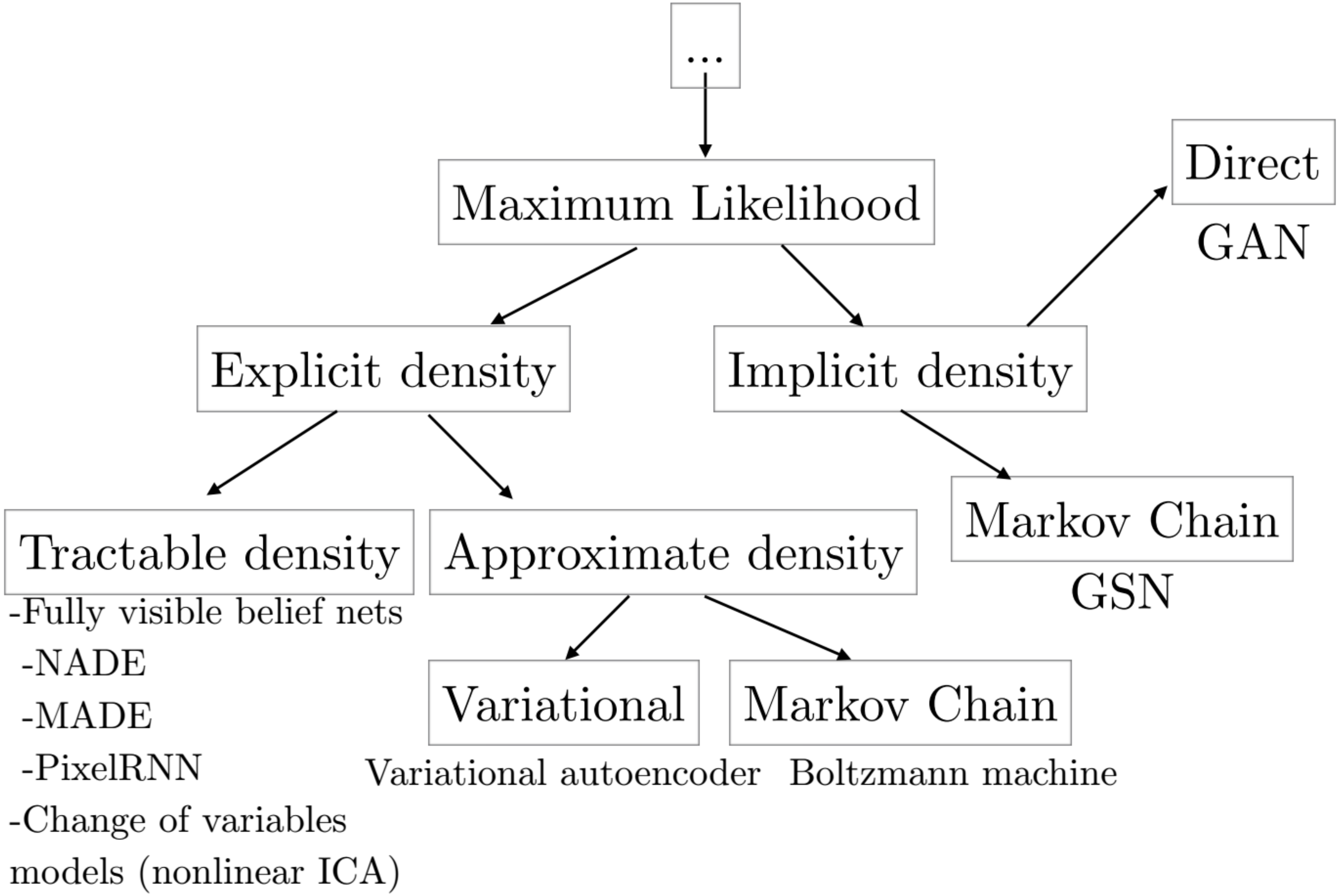
(b) Generated by DCGANs (Reported in [13]).

Applications: Super-resolution



Applications: Cross-model translation

A map of generative models

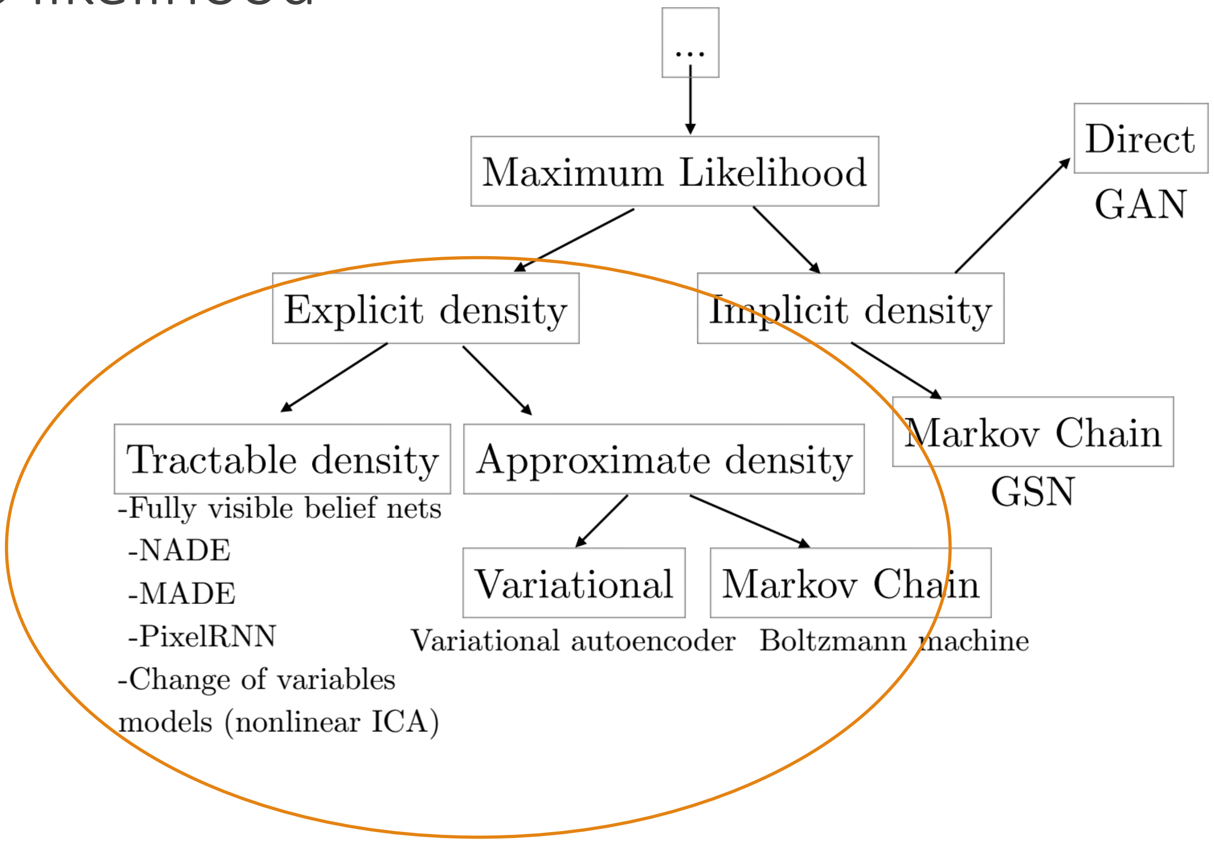


Explicit density models

- Plug in the model density function to likelihood
- Then maximize the likelihood

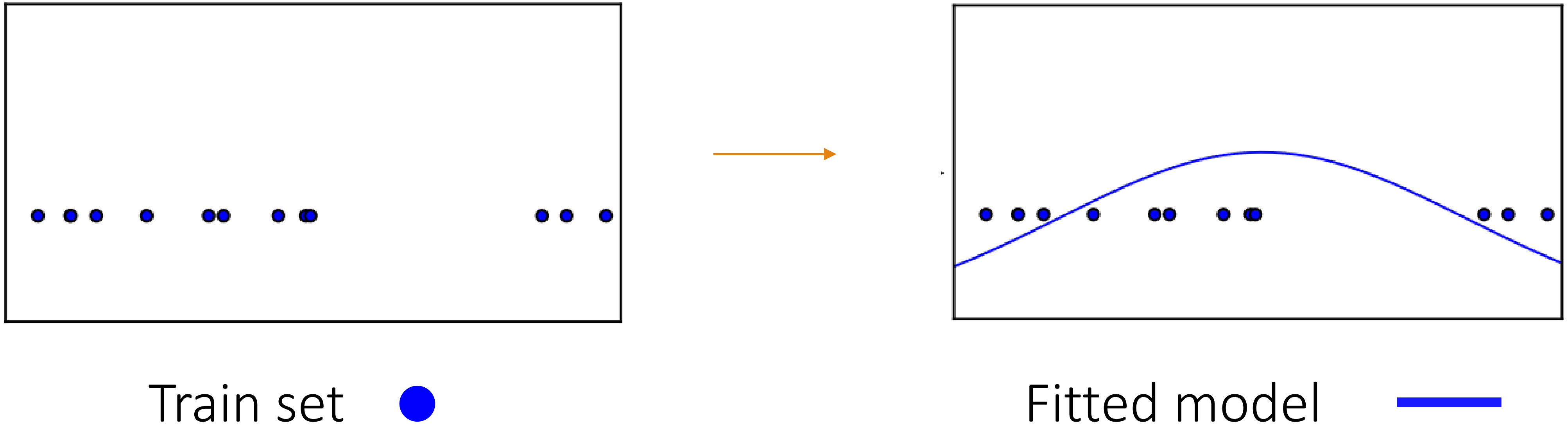
Problems

- Modes must be <u>complex enough</u> → to match data complexity
- Also, model must be <u>computationally tractable</u>
- More details in the next lectures



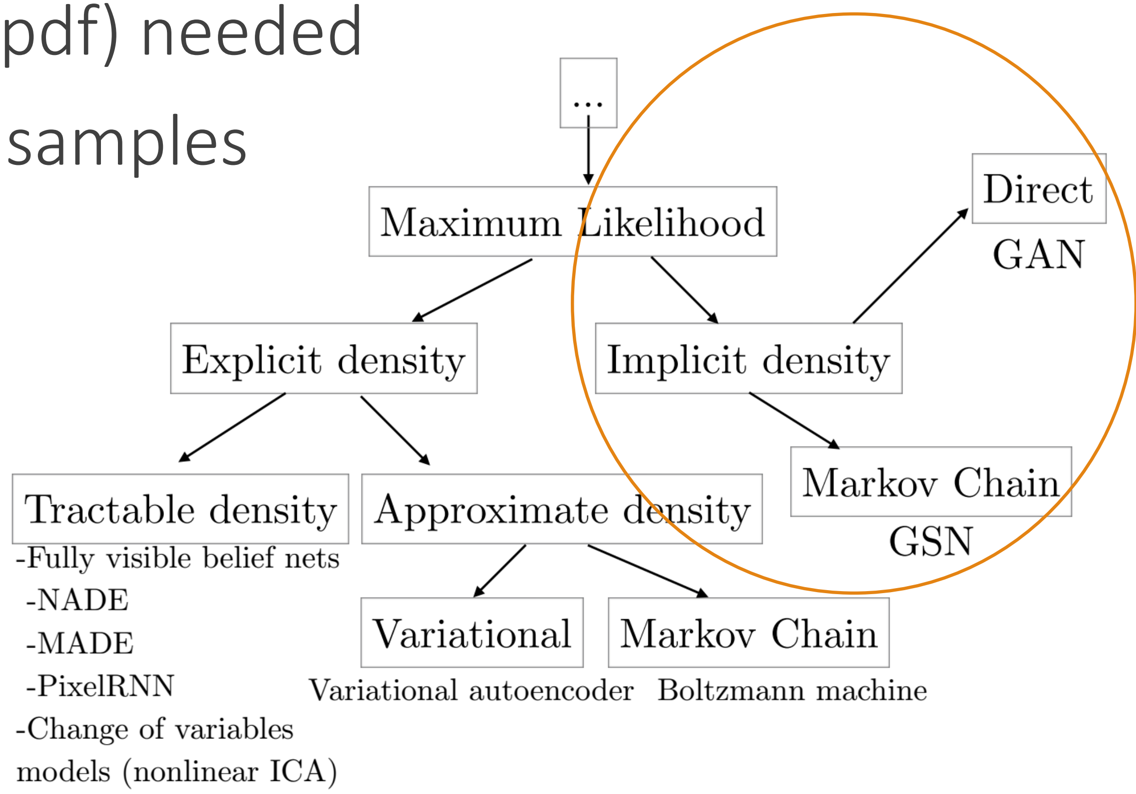
Generative modeling: Case I

- Density estimation



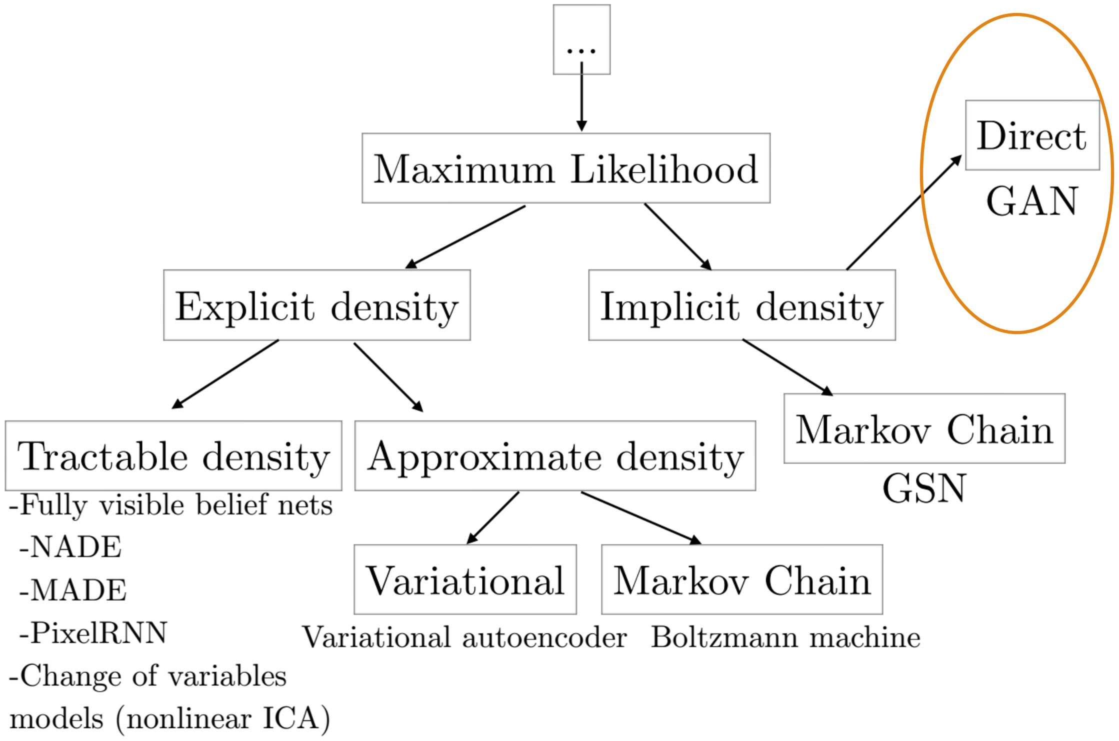
Implicit density models

- No explicit probability density function (pdf) needed
- Instead, a sampling mechanism to draw samples from the pdf without knowing the pdf



Implicit density models: GANs

- Sample data in parallel
- Few restrictions on generator model
- No Markov Chains needed
- No variational bounds
- Better qualitative examples
  - Weak but true



Generative modeling: Case II

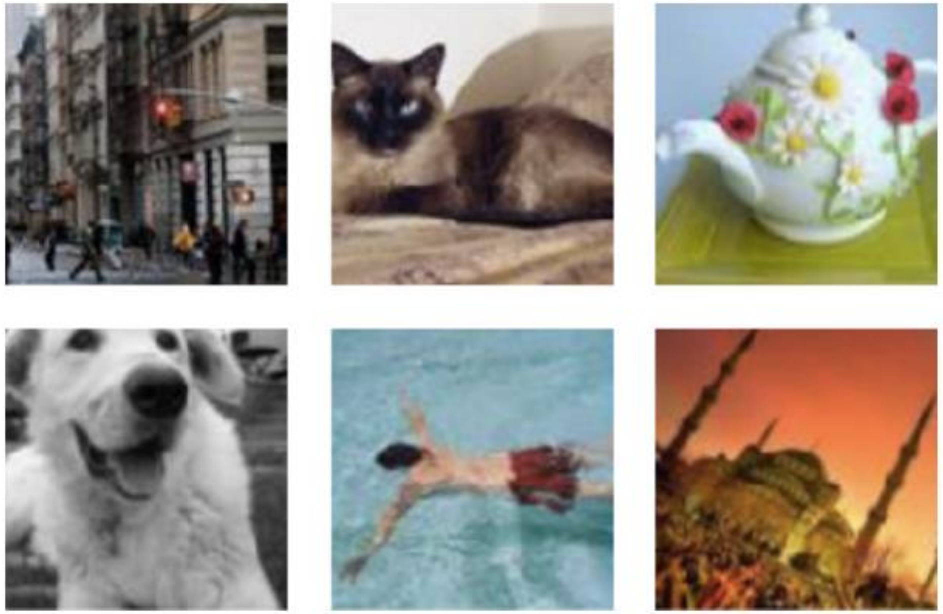
- Sample Generation

Train examples

Generative modeling: Case II

- Sample Generation

Train examples



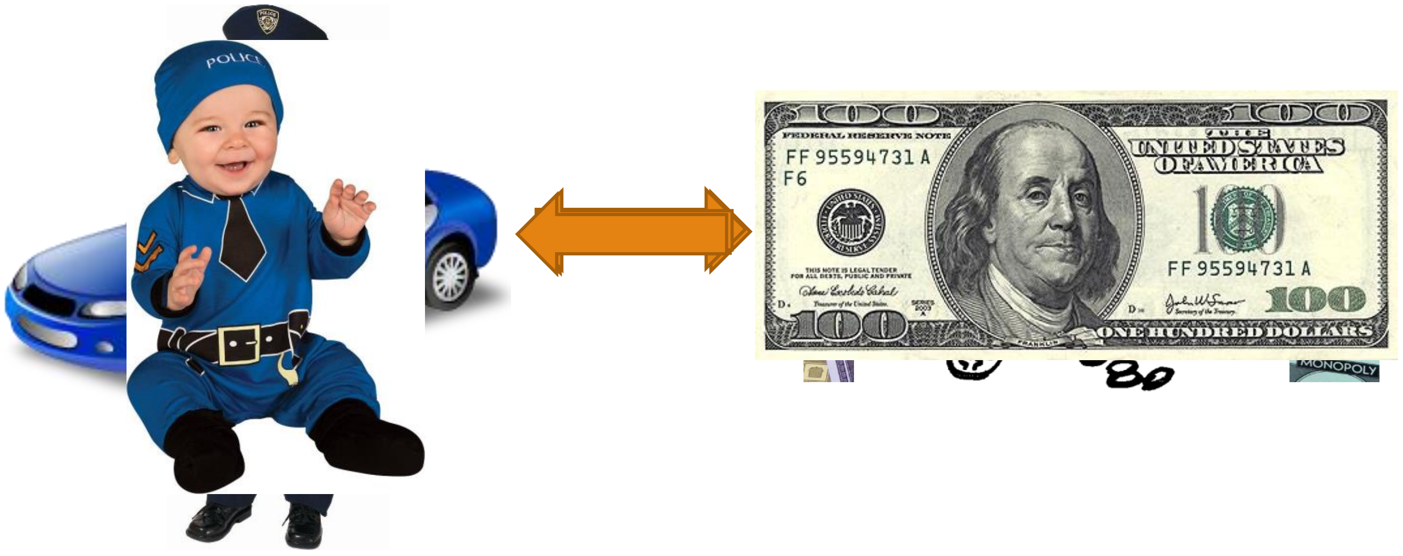
New samples (ideally)

What is a GAN?

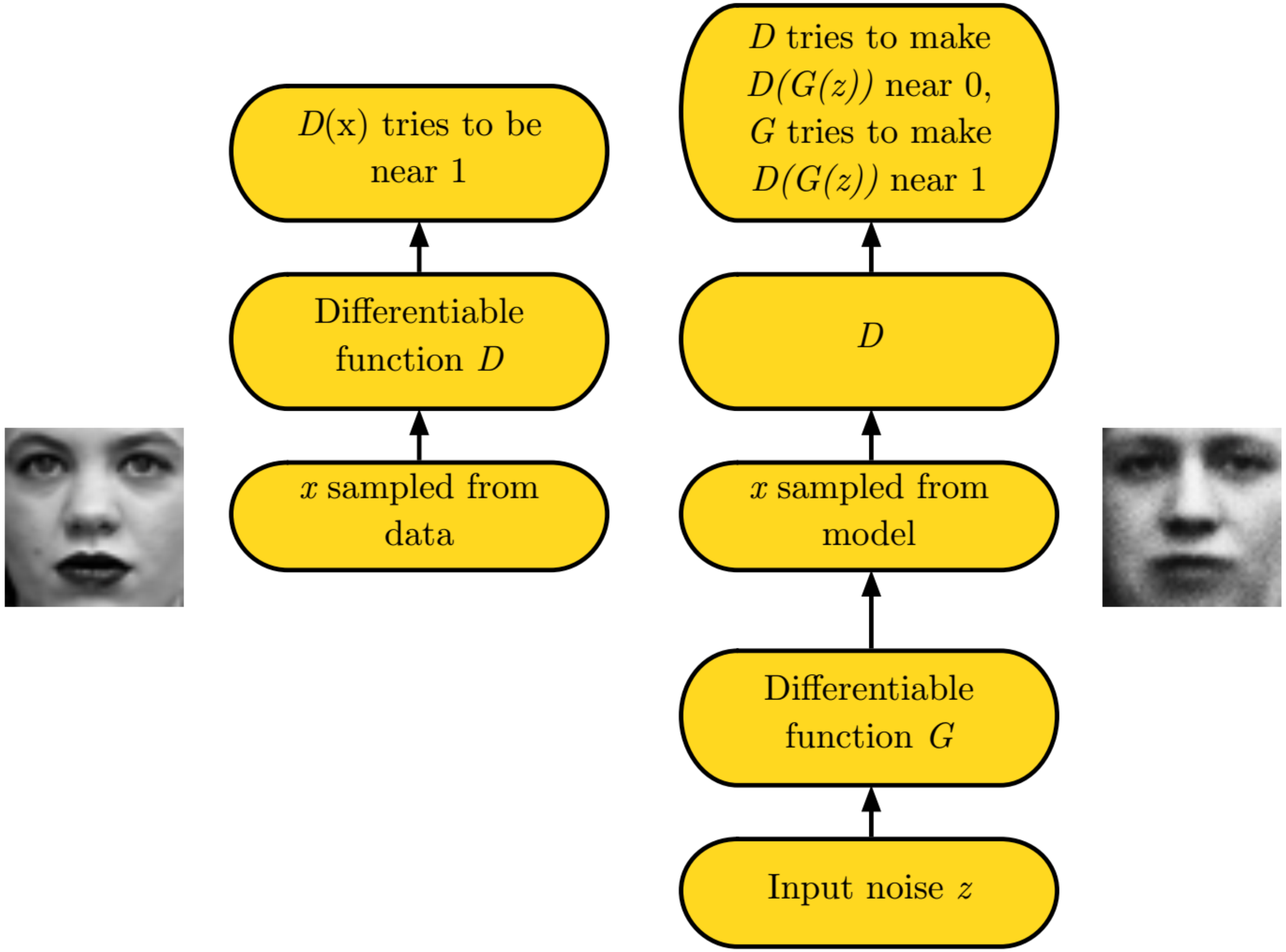
- Generative
  - You can sample novel input samples
  - E.g., you can literally “create” images that never existed
- Adversarial
  - Our generative model G learns adversarially, by fooling an discriminative oracle model D
- Network
  - Implemented typically as a (deep) neural network
  - Easy to incorporate new modules
  - Easy to learn via backpropagation

GAN: Intuition

- Assume you have two parties
  - Police: wants to recognize fake money as reliably as possible
  - Counterfeiter: wants to make as realistic fake money as possible
- The police forces the counterfeiter to get better (and vice versa)
- Solution relates to Nash equilibrium

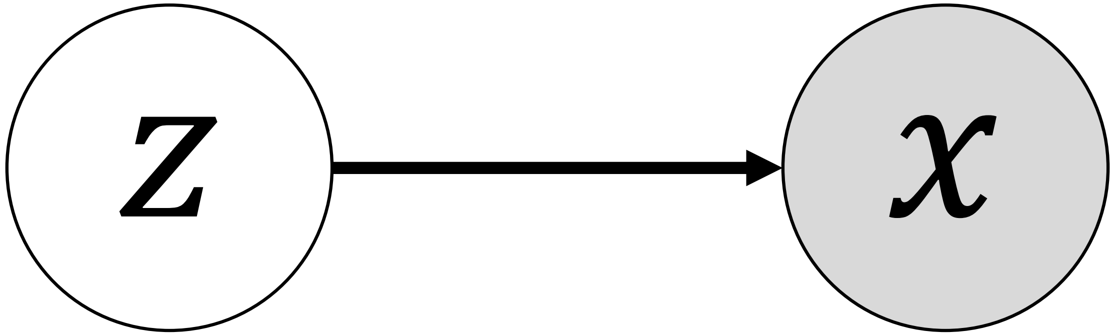


GAN: Pipeline



Generator network $x = G(z; \theta^{(\mathrm{G})})$

- Must be differentiable
- No invertibility requirement
- Trainable for any size of z
- Can make conditionally Gaussian given z, but no strict requirement



Generator & Discriminator: Implementation

- The discriminator is just a standard neural network
- The generator looks like an inverse discriminator

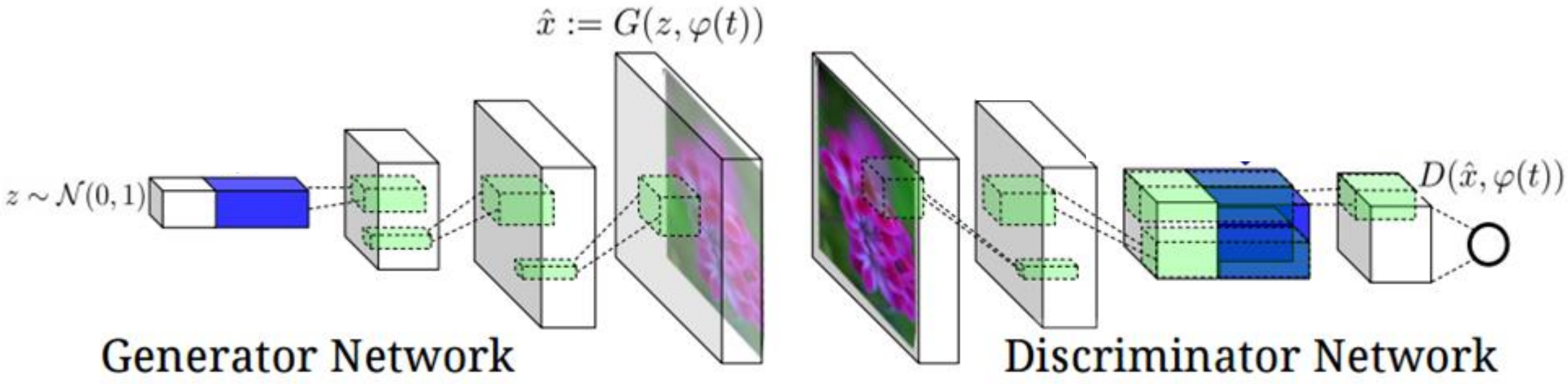


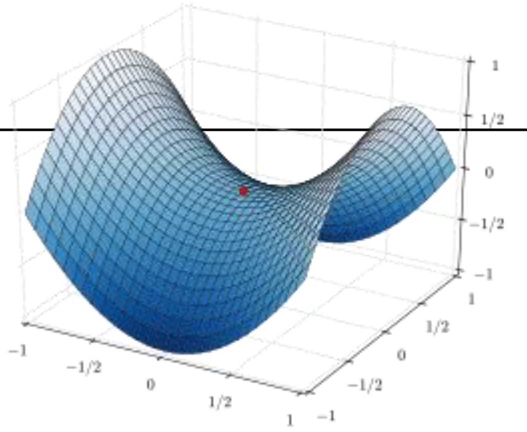
Figure 2. Our text-conditional convolutional GAN architecture. Text encoding $\varphi(t)$ is used by both generator and discriminator. It is projected to a lower-dimensions and depth concatenated with image feature maps for further stages of convolutional processing.

Network Architecture

Training definitions

- Minimax
- Maximin
- Heuristic, non-saturating game
- Max likelihood game

Minimax Game



- $J^{(D)} = -\frac{1}{2}\mathbb{E}_{x\sim p_{data}} \log D(x) \; - \frac{1}{2}\mathbb{E}_{z\sim p_z} \log(1 - D(G(z)))$

- $D(x) = 1 \rightarrow$ The discriminator believes that x is a true image
- $D(G(z)) = 1 \rightarrow$ The discriminator believes that $G(z)$ is a true image

- Equilibrium is a saddle point of the discriminator loss
- Final loss resembles Jenssen-Shannon divergence

[NIPS 2016 Tutorial: Generative Adversarial Networks]

A reasonable loss for the generator?

Minimax Game

- For the simple case of zero-sum game

$$J^{(G)} = -J^{(D)}$$

- So, we can summarize game by

$$V\left(\theta^{(\mathrm{D})}, \theta^{(\mathrm{G})}\right) = -J^{(D)}(\theta^{(\mathrm{D})}, \theta^{(\mathrm{G})})$$

- Easier theoretical analysis

- In practice not used → when the discriminator starts to recognize fake samples, then …

Minimax Game

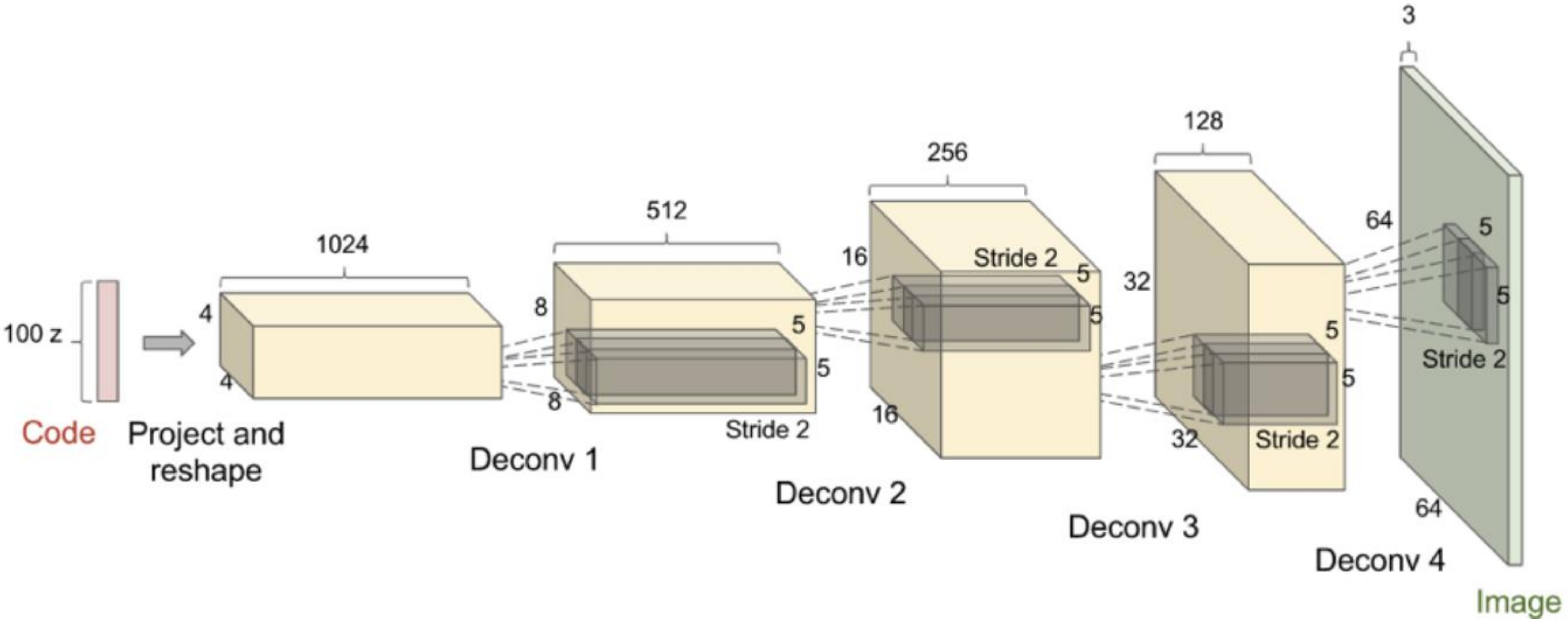
- For the simple case of zero-sum game
$$J^{(G)} = -J^{(D)}$$
- So, we can summarize game by
$$V\left(\theta^{(\mathrm{D})}, \theta^{(\mathrm{G})}\right) = -J^{(D)}(\theta^{(\mathrm{D})}, \theta^{(\mathrm{G})})$$
- Easier theoretical analysis
- In practice not used → when the discriminator starts to recognize fake samples, the generator gradients vanish

Heuristic non-saturating game

- $J^{(D)} = -\frac{1}{2}\mathbb{E}_{x\sim p_{data}} \log D(x) \; - \frac{1}{2}\mathbb{E}_{z\sim p_z} \log(1 - D(G(z))$
- $J^{(G)} = -\frac{1}{2}\mathbb{E}_{z\sim p_z} \log(D(G(z))$

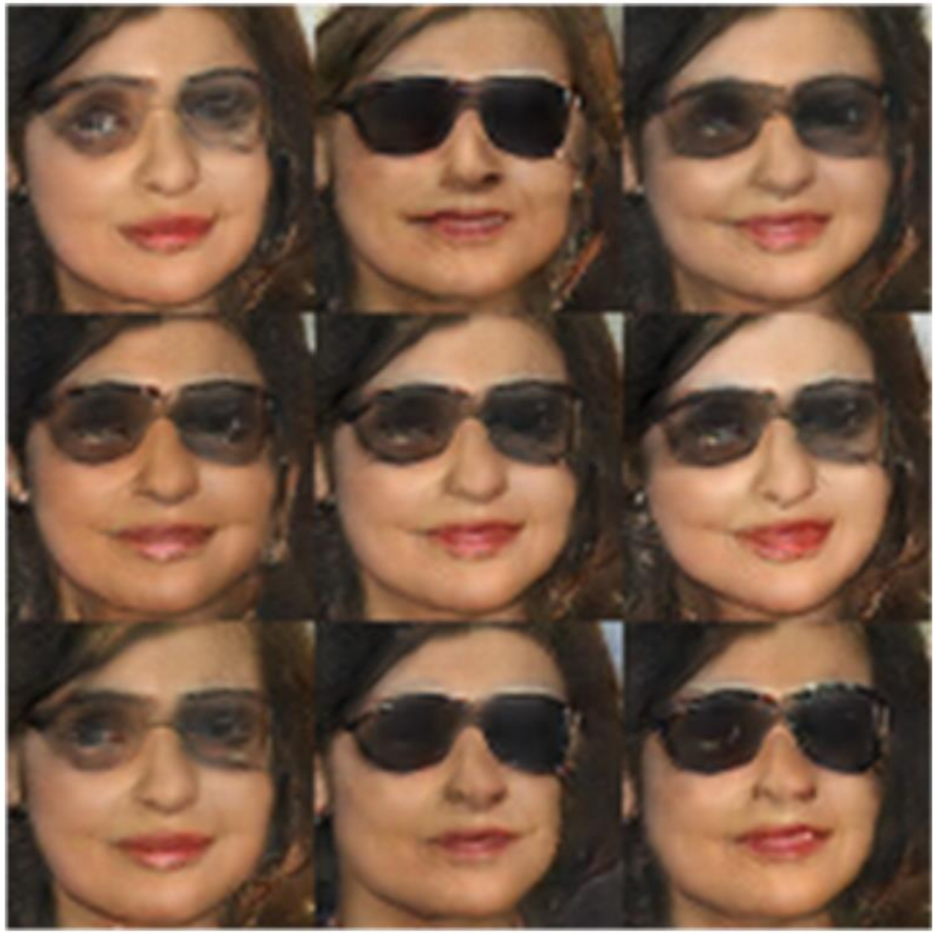
- Equilibrium not any more describable by single loss
- Generator maximizes the log-probability of the discriminator being mistaken
  - Good $G(z)$ → $\mathrm{D}\big(G(z)\big) = 1$ → $J^{(G)}$ is maximized
- Heuristically motivated; generator can still learn even when discriminator successfully rejects all generator samples

DCGAN Architecture



Examples

Even vector space arithmetics …

 -  +  = 

Man with glasses

Man

Woman

Woman with glasses

Modifying GANs for Max-Likelihood

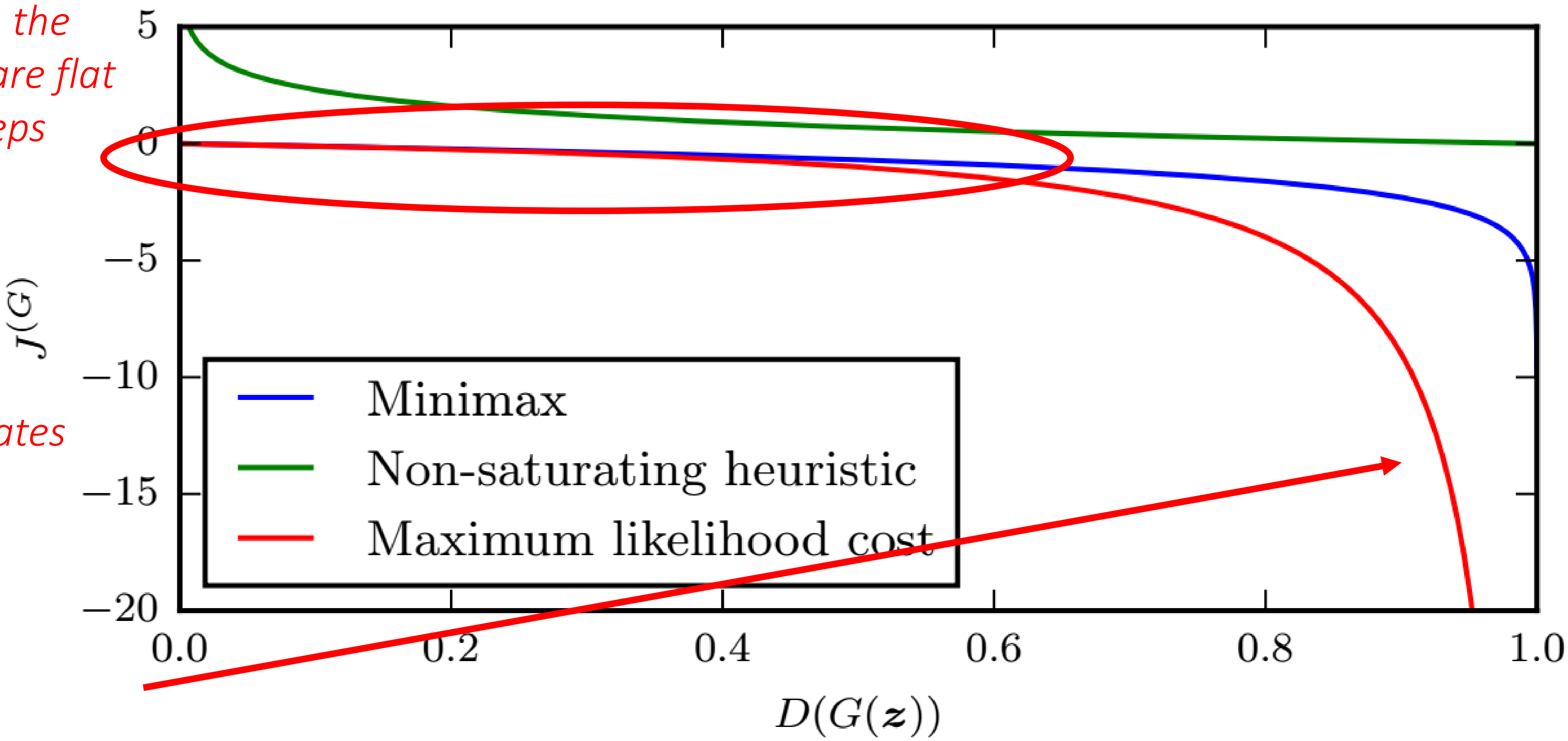
- $J^{(D)} = -\frac{1}{2}\mathbb{E}_{x\sim p_{data}} \log D(x) \; -\frac{1}{2}\mathbb{E}_z \log(1 - D(G(z)))$
- $J^{(G)} = -\frac{1}{2}\mathbb{E}_z \log(\sigma^{-1}(D\big(G(z)\big))$

- When discriminator is optimal, the generator gradient matches that of maximum likelihood

On distinguishability criteria for estimating generative models

Comparison of Generator Losses

When sample is likely fake, the minimax and the ML cost are flat
→ no gradients in early steps



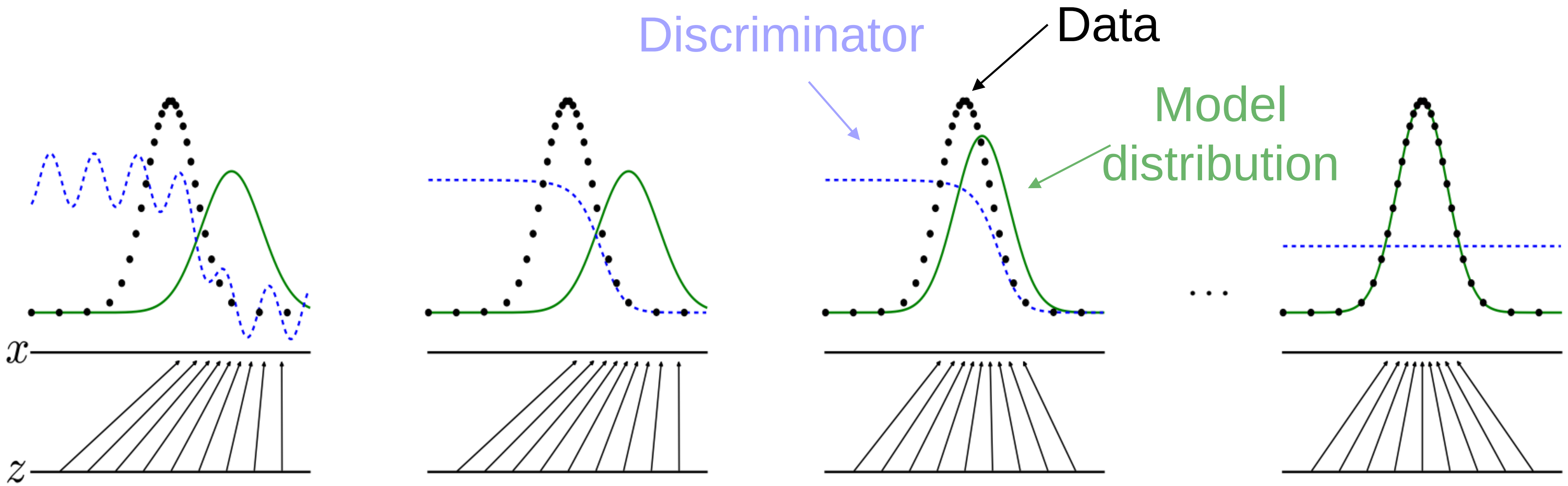
The ML cost variant generates gradients mostly from the "good generations"
→ all gradients from few samples
→ high variance
→ Variance reduction?

Optimal discriminator

- Optimal $D(x)$ for any $p_{data}(x)$ and $p_{model}(x)$ is always

$$D(x) = \frac{p_{data}(x)}{p_{data}(x) + p_{model}(x)}$$

- Estimating this ratio with supervised learning (discriminator) is the key



Why is this the optimal discriminator?

- $L(D, G) = \int_x p_r(x) \log D(x) + p_g(x) \log\big(1 - D(x)\big)\, dx$
  - Minimize $L(D, G)$ w.r.t. $D \rightarrow \frac{dL}{dD} = 0$ and ignore the integral (we sample over all x)
  - The function $x \rightarrow a \log x + b \log(1 - x)$ attains max in $[0, 1]$ at $\frac{a}{a+b}$
- The optimal discriminator

$$D^*(x) = \frac{p_r(x)}{p_r(x) + p_g(x)}$$

  - And at **optimality** $p_g(x) \rightarrow p_r(x)$, thus

$$D^*(x) = \frac{1}{2}$$
$$L(G^*, D^*) = -2 \log 2$$

GANs and Jensen-Shannon divergence

- By expanding the Jensen-Shannon divergence, we have

$$D_{JS}(p_r||p_g) = \frac{1}{2} D_{KL}(p_r|| \frac{p_r + p_g}{2}) + \frac{1}{2} D_{KL}(p_g|| \frac{p_r + p_g}{2})$$

$$= \frac{1}{2}\left(\log 2 + \int_x p_r(x) \log \frac{p_r(x)}{p_r(x) + p_g(x)} \, dx + \log 2 \right.$$

GANs and Jensen-Shannon divergence

- By expanding the Jensen-Shannon divergence, we have

$$D_{JS}(p_r||p_g) = \frac{1}{2} D_{KL}(p_r||\frac{p_r + p_g}{2}) + \frac{1}{2} D_{KL}(p_g||\frac{p_r + p_g}{2})$$
$$= \frac{1}{2}\left(\log 2 + \int_x p_r(x) \log \frac{p_r(x)}{p_r(x) + p_g(x)} dx + \log 2\right.$$

https://lilianweng.github.io/lil-log/2017/08/20/from-GAN-to-WGAN.html

Is the divergence important?

- Does the divergence make a difference?
- Is there a difference between KL-divergence, Jensen-Shannon divergence, …

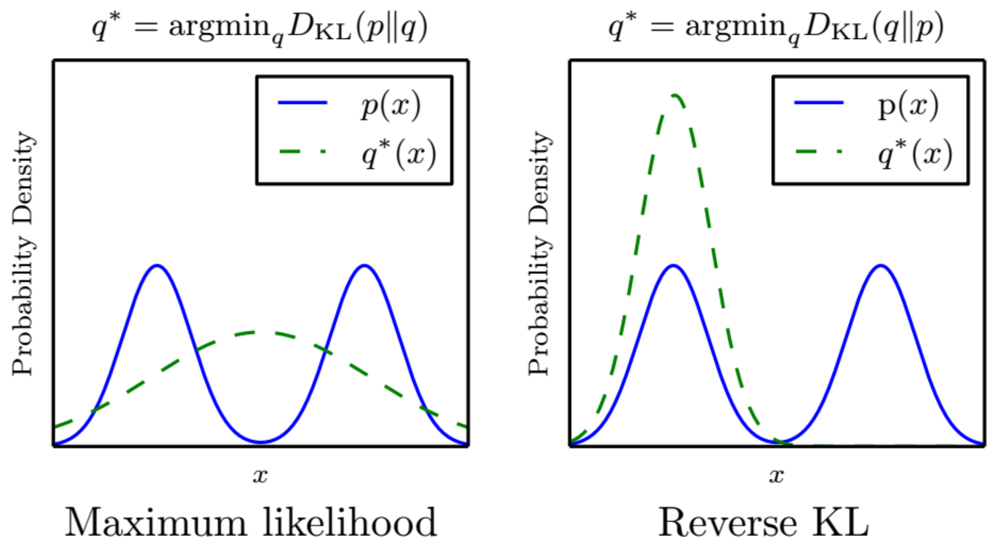
$$D_{KL}(p_r||p_g) = \int_x p_r \log \frac{p_r}{p_g} dx$$

$$D_{JS}(p_r||p_g) = \frac{1}{2} D_{KL}(p_r||\frac{p_r + p_g}{2}) + \frac{1}{2} D_{KL}(p_g||\frac{p_r + p_g}{2})$$

- Let’s check the KL-divergence

Is the divergence important?

- Forward KL divergence: $D_{KL}(p(x)||q^*(x)) \rightarrow$ high probability everywhere that the data occurs
- Backward KL divergence: $D_{KL}(q^*(x)||p(x)) \rightarrow$ low probability wherever the data does not occur
- Which version makes the model "conservative"?

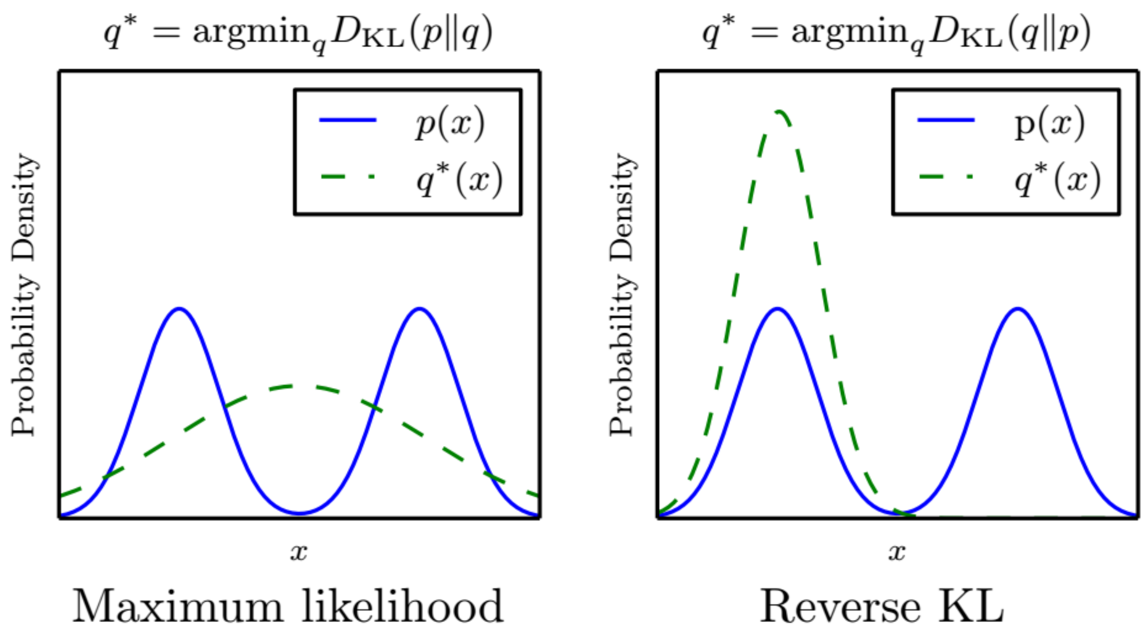


p_r is what we get and cannot change
p_g is what we make through our model and (through training) change

$$D_{KL}(p_r||p_g) = \int_x p_r \log \frac{p_r}{p_g} dx$$

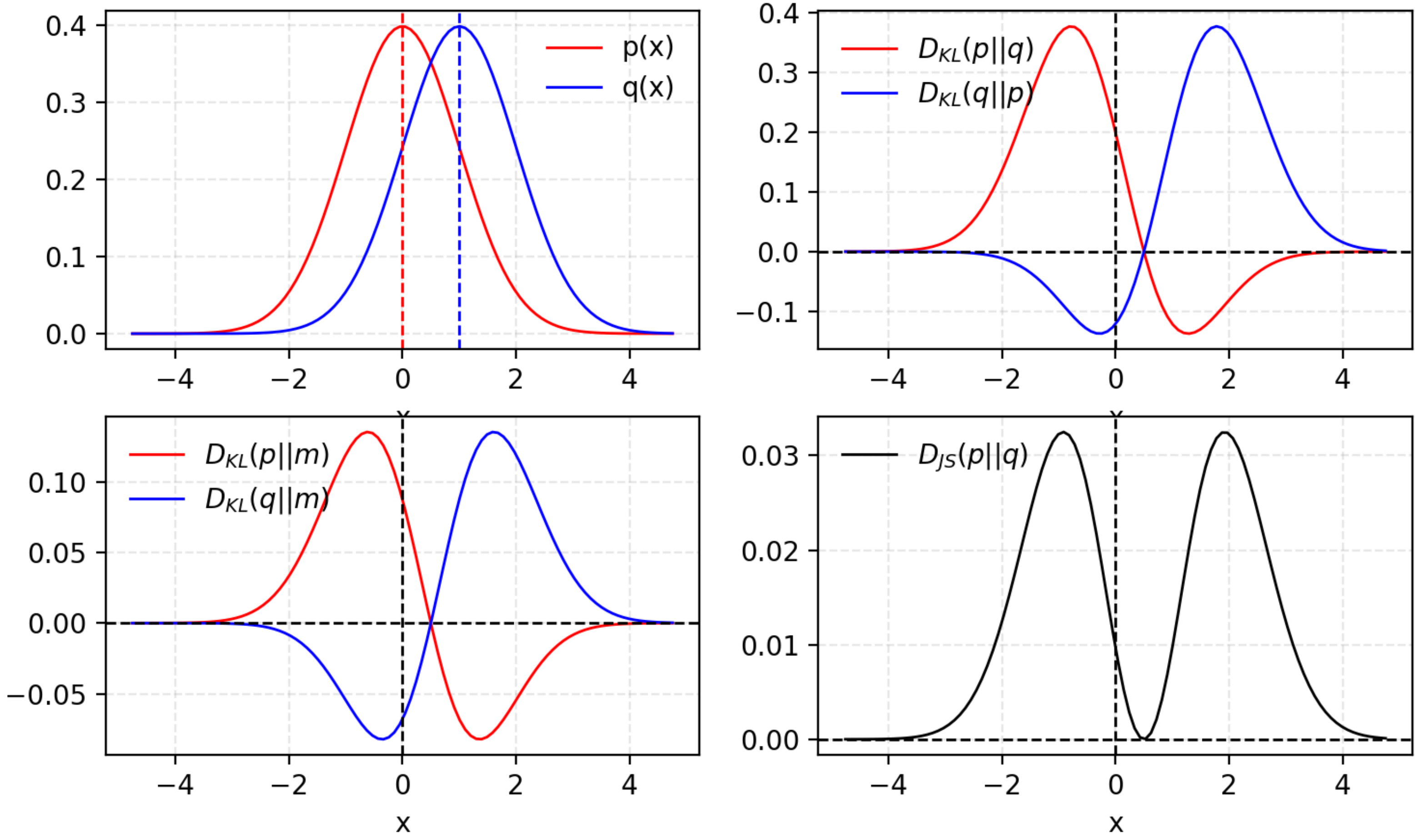
Is the divergence important?

- $D_{KL}(p(x)||q^*(x))$→ high probability everywhere that the data occurs
- $D_{KL}(q^*(x)||p(x))$ → low probability wherever the data does not occur
- Which version makes the model "conservative"?
- $D_{KL}(q^*(x)||p(x)) = \int q^*(x)\log\frac{q^*(x)}{p(x)}$
  - Avoid areas where $p(x) \rightarrow 0$
- Backward KL → Zero-forcing
  - $q^*(x) \rightarrow 0$ in areas when approximation $\frac{q^*(x)}{p(x)}$ cannot be good



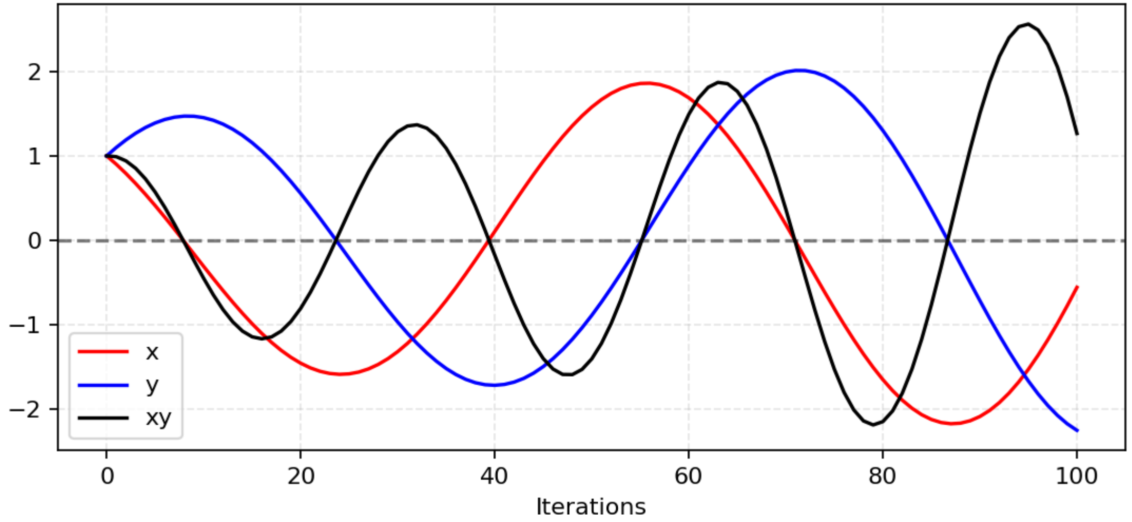
KL vs JS

- JS is symmetric, KL is not



GAN Problems: Reaching Nash equilibrium causes instabilities

- GANs is a mini-max optimization
  - Non-cooperative game with a tied objective
- Training is not always easy
  → When optimizing one player/network, we might hurt the other one
  → oscillations
- Assume two players $f(x) = xy$
  We optimize one step at a time
  - Player 1 minimizes: $\min_x f_1(x) = xy \Rightarrow \frac{df_1}{dx} = y$
    $\Rightarrow x_{t+1} = x_t - \eta \cdot y$
  - Player 2 minimizes: $\min_y f_2(x) = -xy \Rightarrow \frac{df_2}{dx} = -x$
    $\Rightarrow y_{t+1} = y_t + \eta \cdot x$

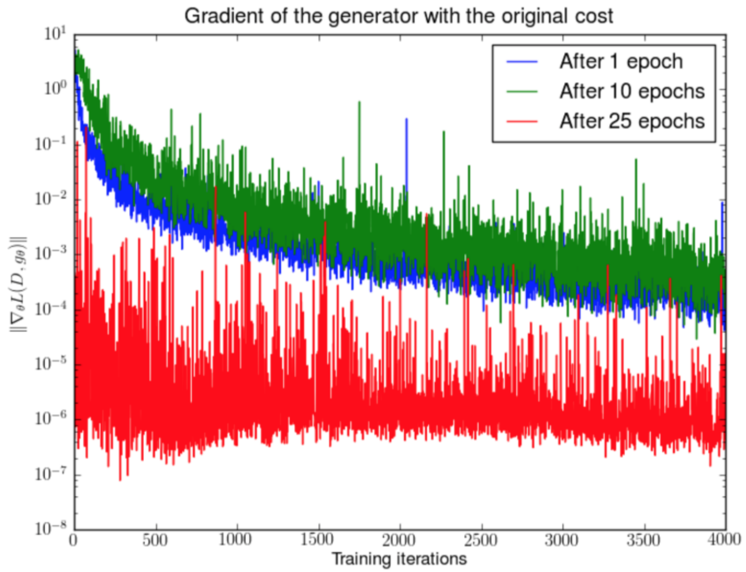


https://lilianweng.github.io/lil-log/2017/08/20/from-GAN-to-WGAN.html

GAN Problems: Vanishing Gradients

$$J^{(D)} = -\frac{1}{2}\mathbb{E}_{x\sim p_{data}} \log D(x) \ -\frac{1}{2}\mathbb{E}_z \log(1 - D(G(z)))$$

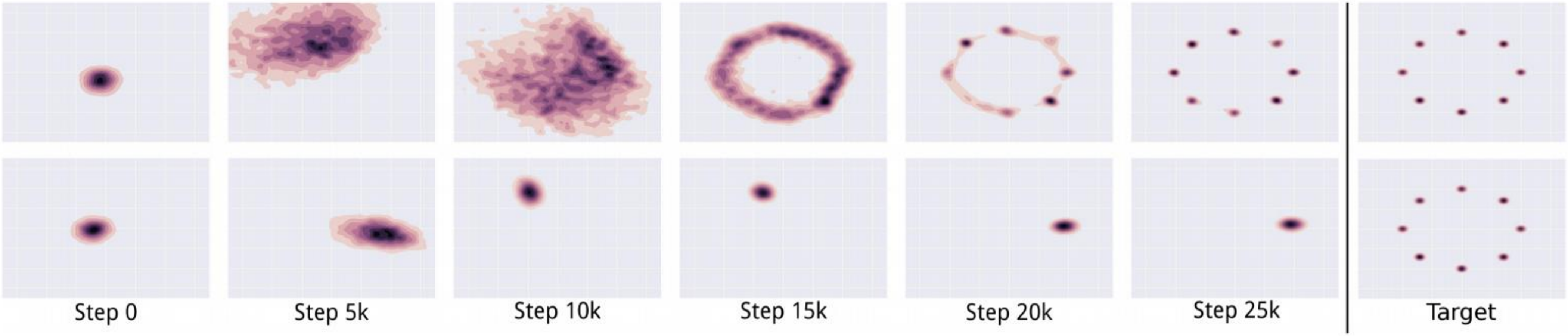
$$J^{(G)} = -\frac{1}{2}\mathbb{E}_z \log(D(G(z)))$$



- If the discriminator is quite bad
  → no accurate feedback for generator
  → no reasonable generator gradients
- But, if the discriminator is perfect, $D(x) = D^*(x)$
  → gradients go to 0
  → no learning anymore
- Bad when this happens early in the training
  - Easier to train the discriminator than the generator

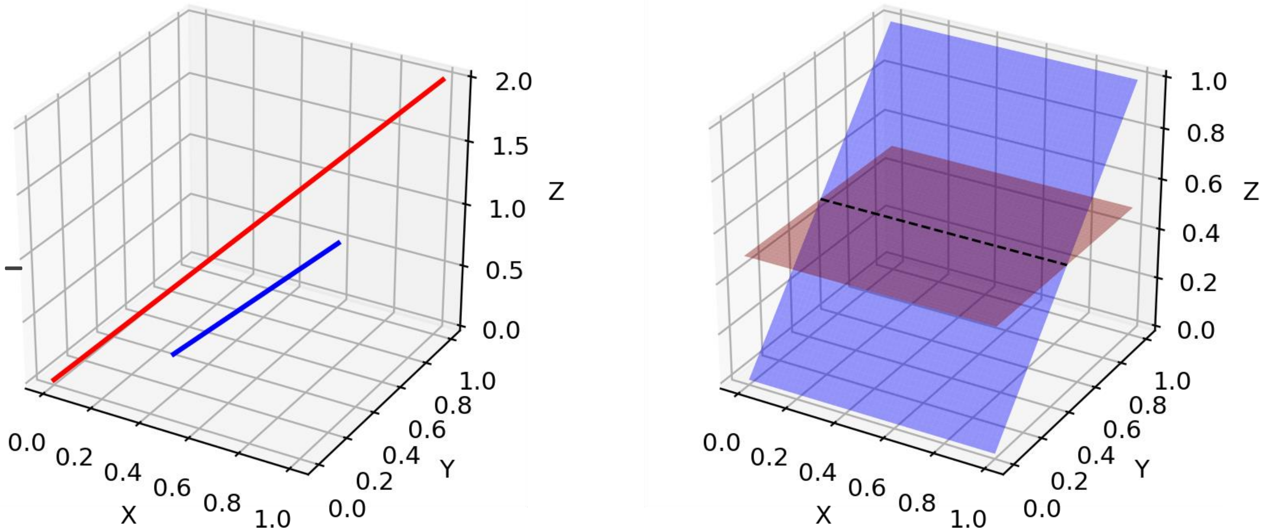
GAN Problems: Mode collapse

- Very low variability
- It is safer for the generator to produce samples from the mode it knows it approximates well



GAN Problems: Low dimensional supports

- Data lie in low-dim manifolds
- However, the manifold is not known
- During training p_g is not perfect either, especially in the start
- So, the support of p_r and p_g is non-overlapping and disjoint
  → not good for KL/JS divergences
- Easy to find a discriminating line

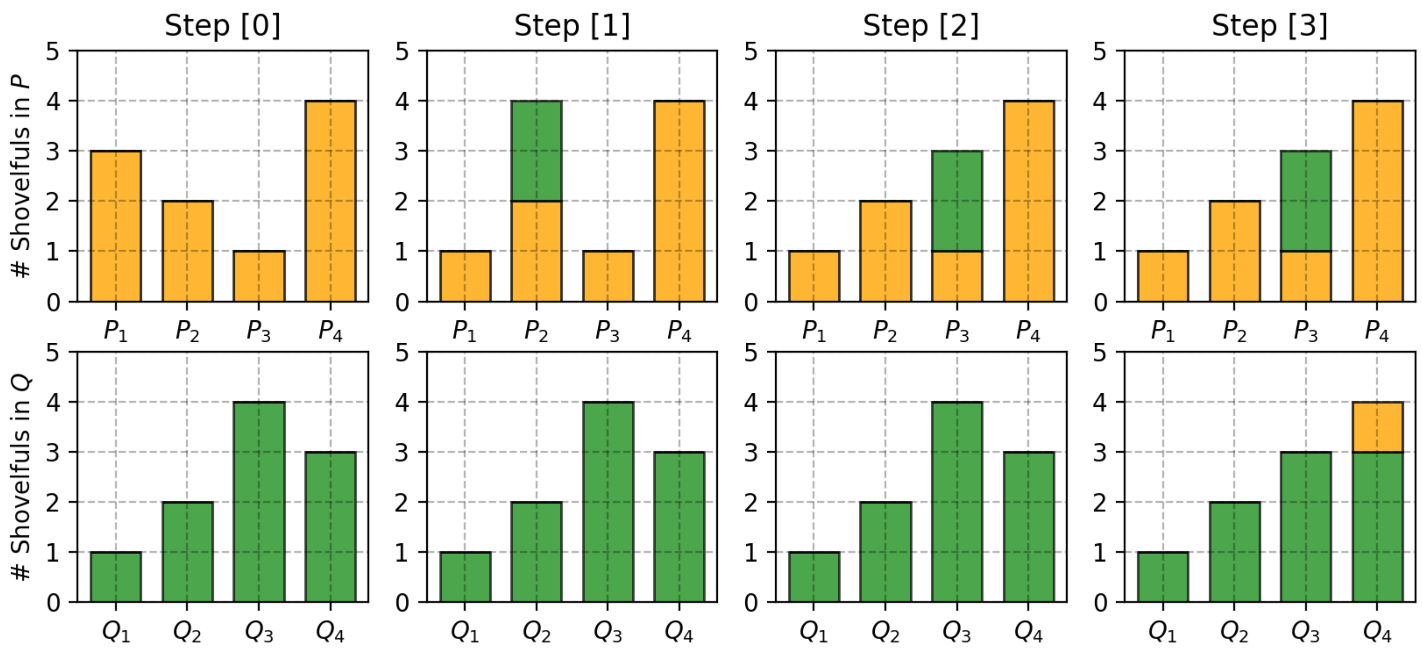


Wasserstein GAN

- Instead of KL/JS, use Wasserstein (Earth Mover's) Distance

$$W(p_r, p_g) = \inf_{\gamma \sim \Pi(\mathrm{p_r}, \mathrm{p_g})} \mathrm{E}_{(\mathrm{x},\mathrm{y}) \sim \gamma} |x - y|$$

- Even for non-overlapping supports, the distance is meaningful



Feature matching

- Instead of matching image statistics, match feature statistics

$$J^{(D)} = \left\| \mathbb{E}_{x \sim p_r} f(x) - \mathbb{E}_{z \sim p_z} f\big(G(z)\big) \right\|_2^2$$

- f can be any statistic of the data, like the mean or the median

Training procedure

- Use SGD-like algorithm of choice
  - Adam Optimizer is a good choice
- Use two mini-batches simultaneously
  - The first mini-batch contains real examples from the training set
  - The second mini-batch contains fake generated examples from the generator
- Optional: run k-steps of one player (e.g. discriminator) for every step of the other player (e.g. generator)

Use labels if possible

- Learning a conditional model $p(y|x)$ is often generates better samples
  - Denton et al., 2015
- Even learning $p(x, y)$ makes samples look more realistic
  - Salimans et al., 2016
- Conditional GANs are a great addition for learning with labels

One-sided label smoothing

- Default discriminator cost:

```
cross_entropy(1., discriminator(data))
+ cross_entropy(0., discriminator(samples))
```

- One-sided label smoothing:

```
cross_entropy(0.9, discriminator(data))
+ cross_entropy(0., discriminator(samples))
```

- Do not smooth negative labels:

```
cross_entropy(1.-alpha, discriminator(data))
+ cross_entropy(beta, discriminator(samples))
```

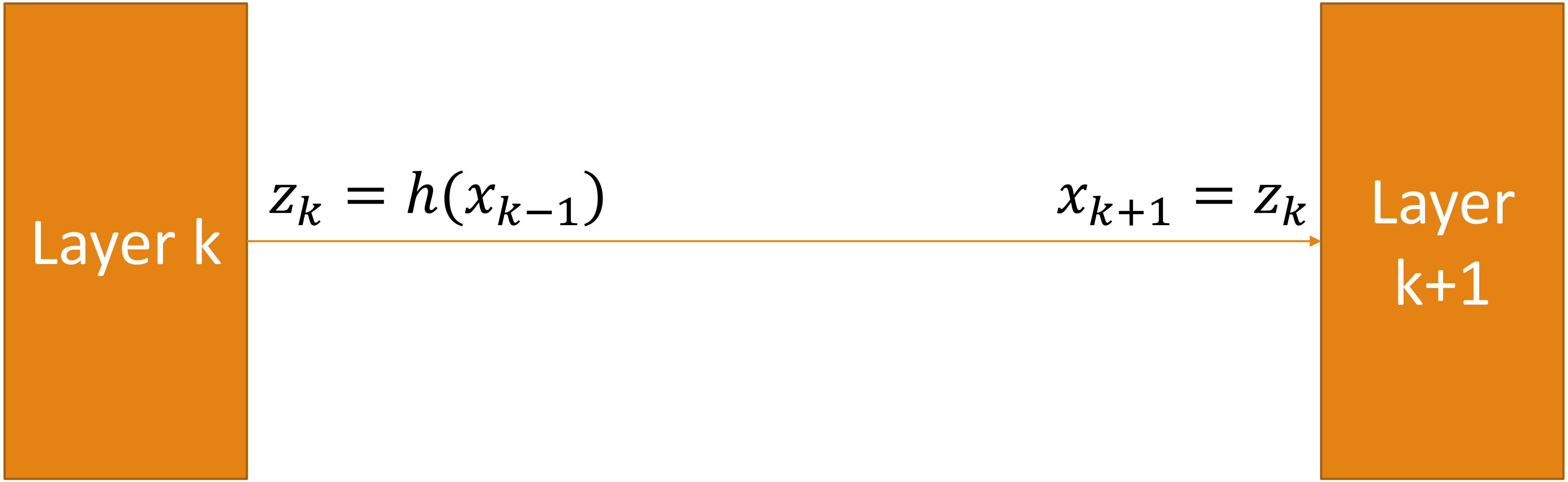
Benefits of label smoothing

- Max likelihood often is overconfident
  - Might return accurate prediction, but too high probabilities
- Good regularizer
  - Szegedy et al., 2015
- Does not reduce classification accuracy, only confidence
- Specifically for GANs
  - Prevents discriminator from giving very large gradient signals to generator
  - Prevents extrapolating to encourage extreme samples

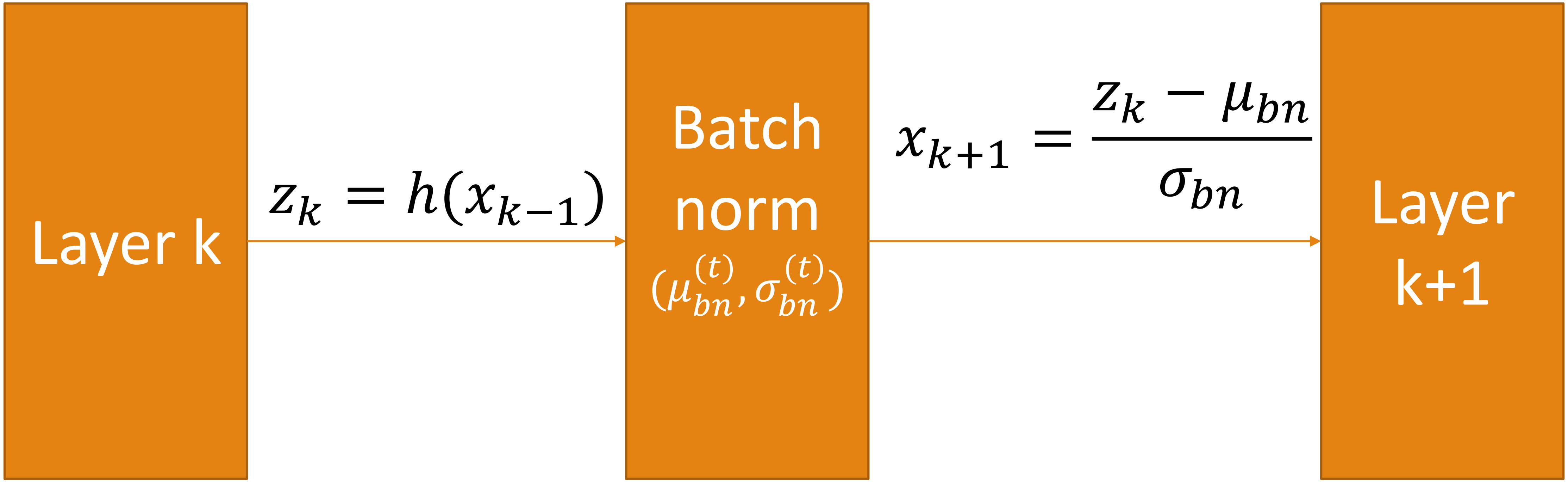
Batch normalization

- Generally, good practice for neural networks
- Given inputs $X = \{x^{(1)}, x^{(2)}, \ldots, x^{(m)}\}$
- Compute mean and standard deviation of features of X: μ_{bn}, σ_{bn}
- Normalize features
  - Subtract mean, divide by standard deviation

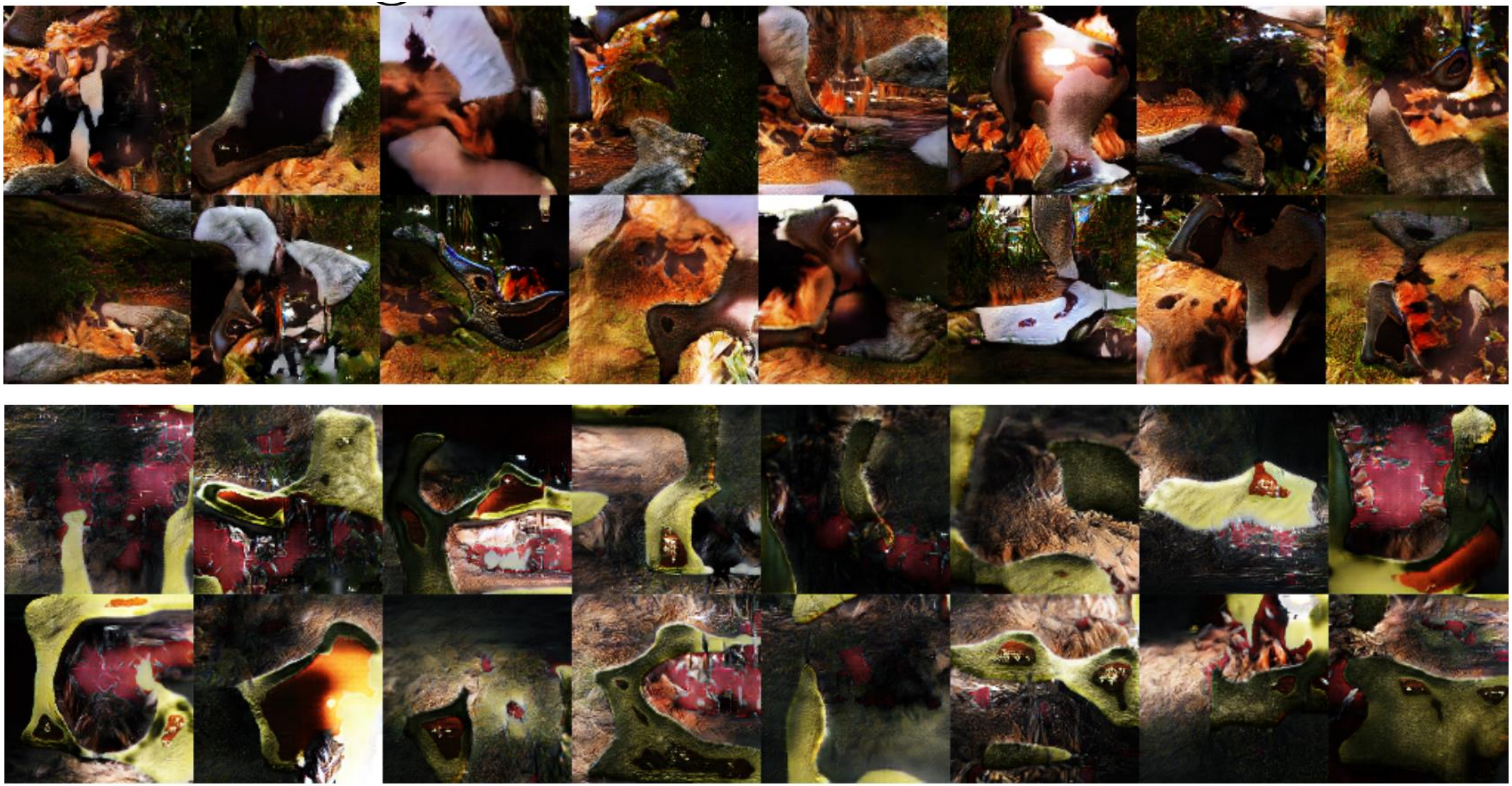
Batch normalization: Graphically



Batch normalization: Graphically



But, can cause strong intra-batch correlation



Reference batch normalization

- Training with two mini-batches
- One fixed reference mini-batch for computing mean and standard deviation
- The other for doing the training as usual
- Proceed as normal, only use the mean and standard deviation for the batch norm from the fixed reference mini-batch
- Problem: Overfitting to the reference mini-batch

	Standard mini-batch	Reference mini-batch
Iteration 1	$\frac{dJ^{(1)}}{d\theta}$	μ_{bn}, σ_{bn}
Iteration 2	$\frac{dJ^{(2)}}{d\theta}$	μ_{bn}, σ_{bn}
Iteration 3	$\frac{dJ^{(3)}}{d\theta}$	μ_{bn}, σ_{bn}

Solution: Virtual batch normalization

- Mini-batch= standard mini-batch + reference/fixed mini-batch

	Standard mini-batch	Reference mini-batch
Iteration 1	$\frac{dJ^{(1)}}{d\theta}$	$\mu_{bn}^{(R)}, \sigma_{bn}^{(R)}$
Iteration 2	$\frac{dJ^{(2)}}{d\theta}$	$\mu_{bn}^{(R)}, \sigma_{bn}^{(R)}$
Iteration 3	$\frac{dJ^{(3)}}{d\theta}$	$\mu_{bn}^{(R)}, \sigma_{bn}^{(R)}$

Balancing Generator & Discriminator

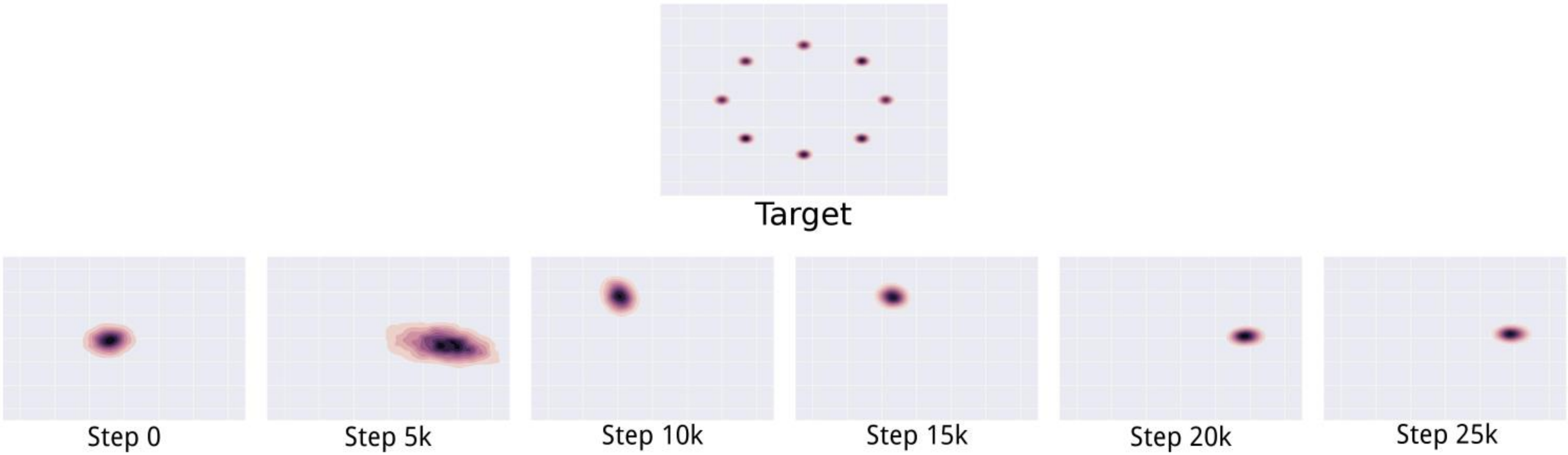
- Usually the discriminator wins
  - That’s good, in that the theoretical justification assume a perfect discriminator
- Usually the discriminator network is bigger than the generator
- Sometimes running discriminator more often than generator works better
  - However, no real consensus
- Do not limit the discriminator to avoid making it too smart
  - Better use non-saturating cost
  - Better use label smoothing

Open Question: Non-convergence

- Optimization is tricky and unstable
  - finding a saddle point does not imply a global minimum
- An equilibrium might not even be reached
- Mode-collapse is the most severe form of non-convergence

Open Question: Mode collapse

- Discriminator converges to the correct distribution
- Generator however places all mass in the most likely point

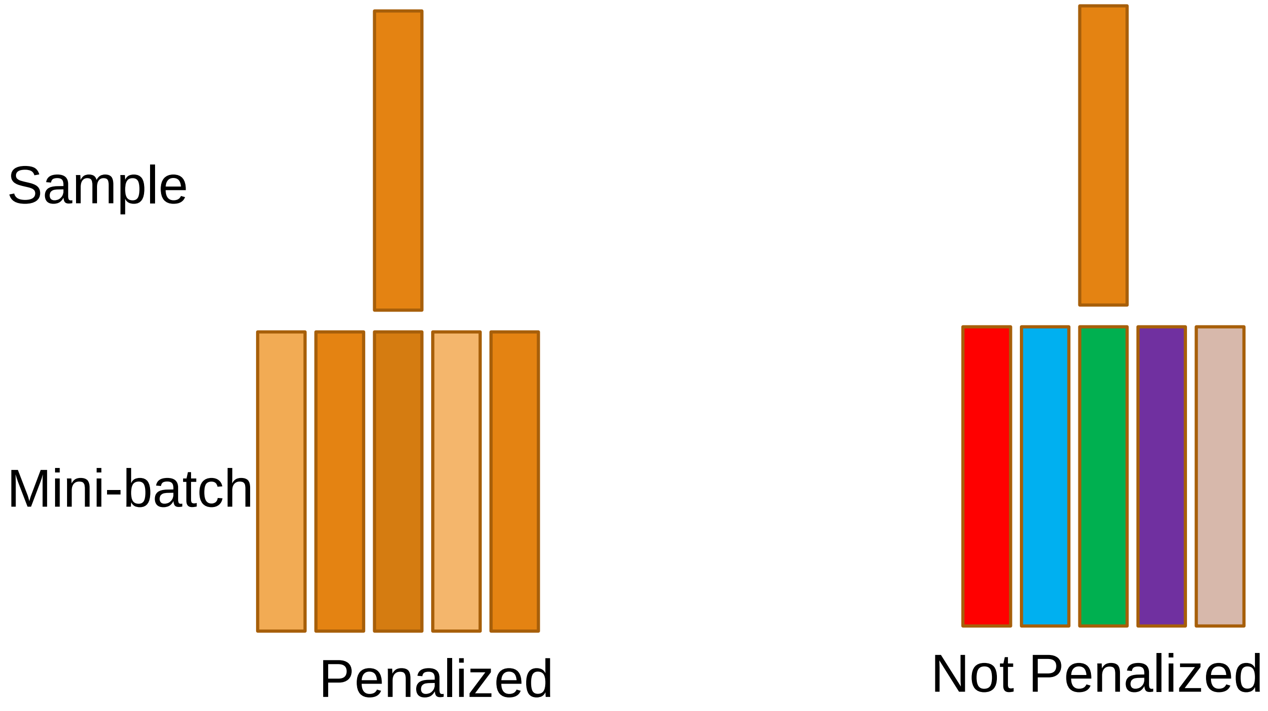


Open Question: Mode collapse

- Discriminator converges to the correct distribution
- Generator however places all mass in the most likely point
- Problem: low sample diversity

Minibatch features

- Classify each sample by comparing to other examples in the mini-batch
- If samples are too similar, the model is penalized



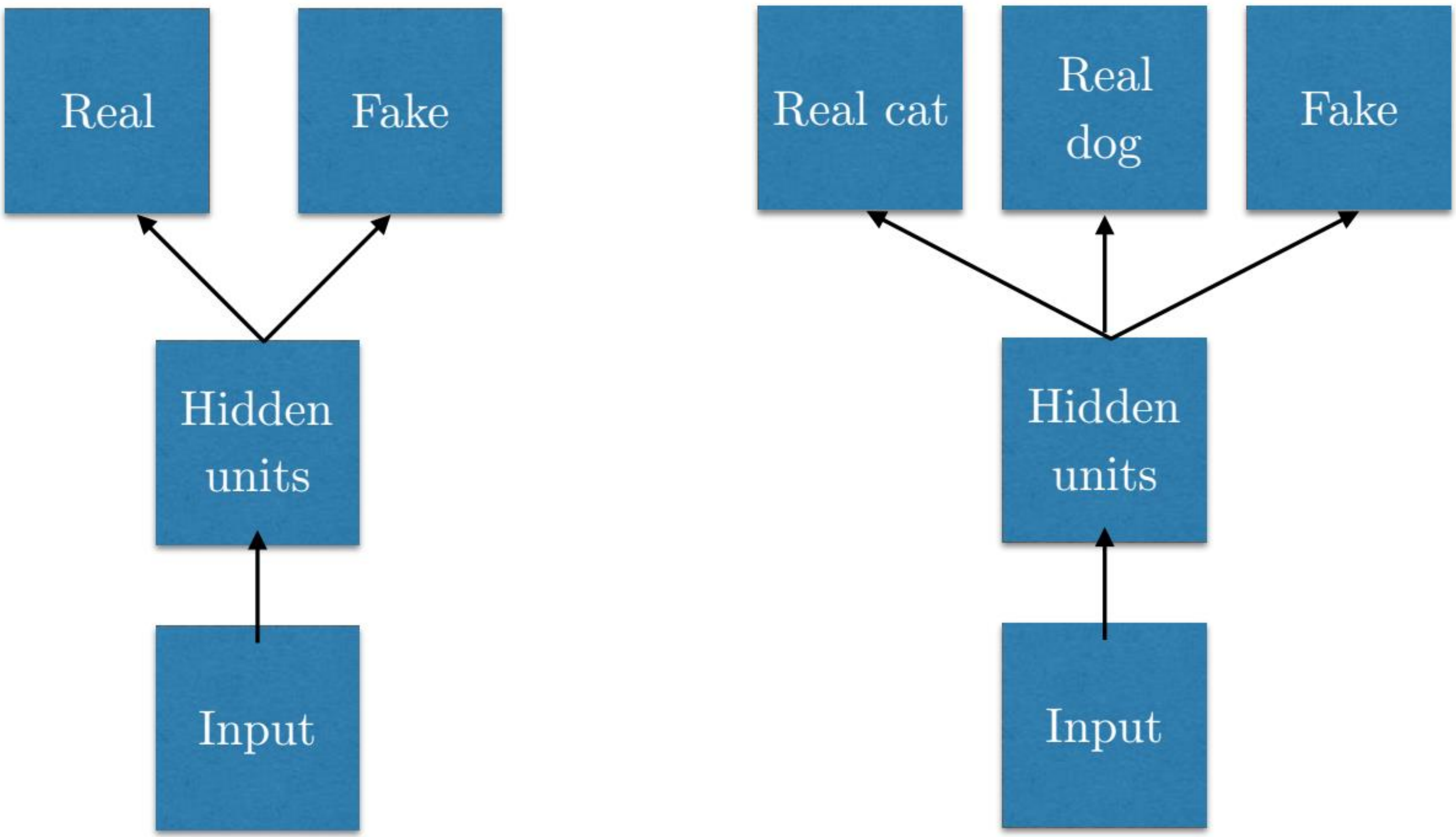
Open Question: Evaluation of GANs

- Despite the nice images, who cares?
- It would be nice to quantitatively evaluate the model
- For GANs it is even hard to estimate the likelihood

Open Question: Discrete outputs

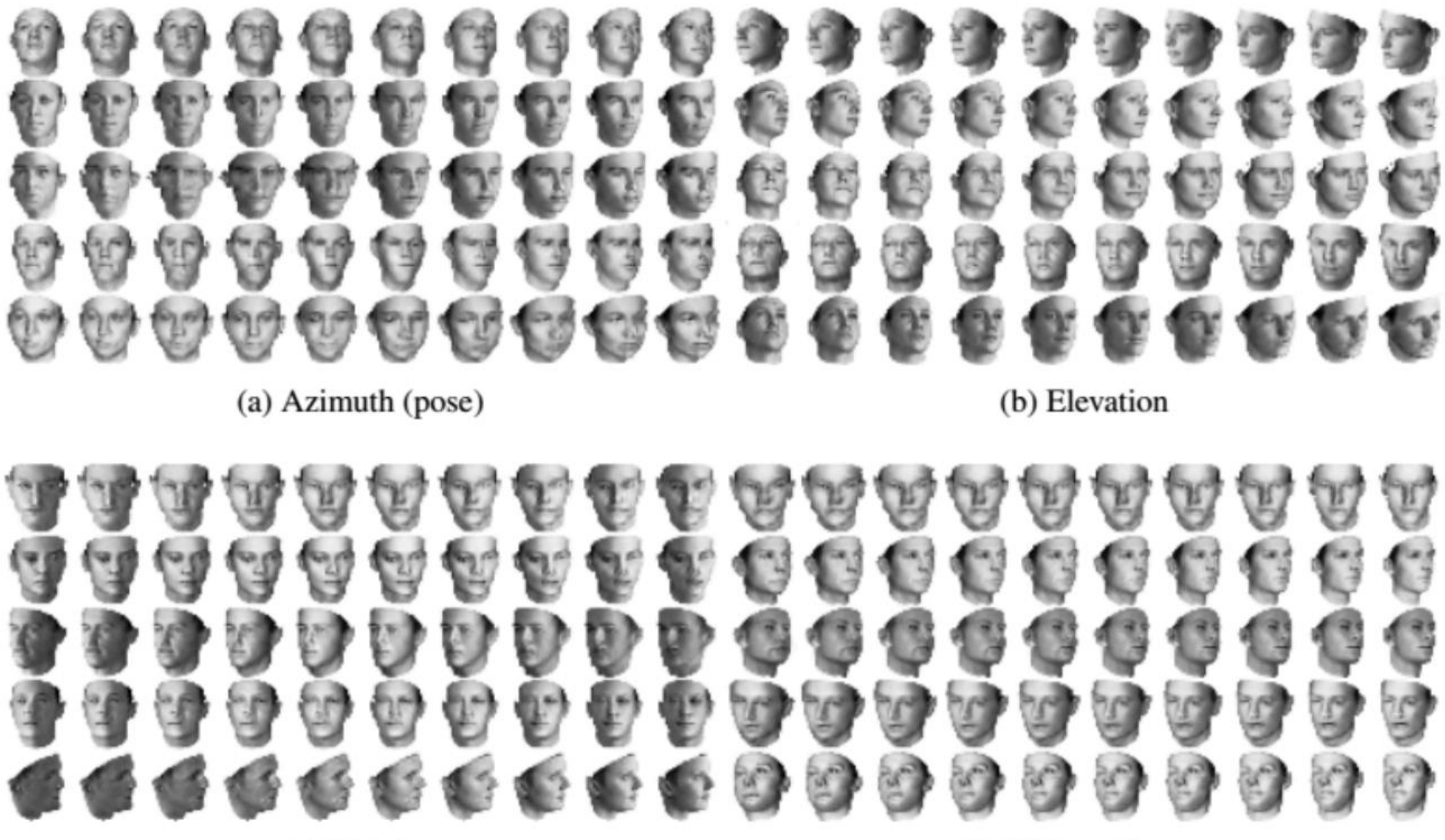
- The generator must be differentiable
- It cannot be differentiable if outputs are discrete
- E.g., harder to make it work for text
- Possible workarounds
  - REINFORCE [Williams, 1992]
  - Concrete distribution [Maddison et al., 2016]
  - Gumbel softmax [Jang et al., 2016]
  - Train GAN to generate continuous embeddings

Open Question: Semi-supervised classification



Interpretable latent codes

- InfoGAN [Chen et al., 2016]

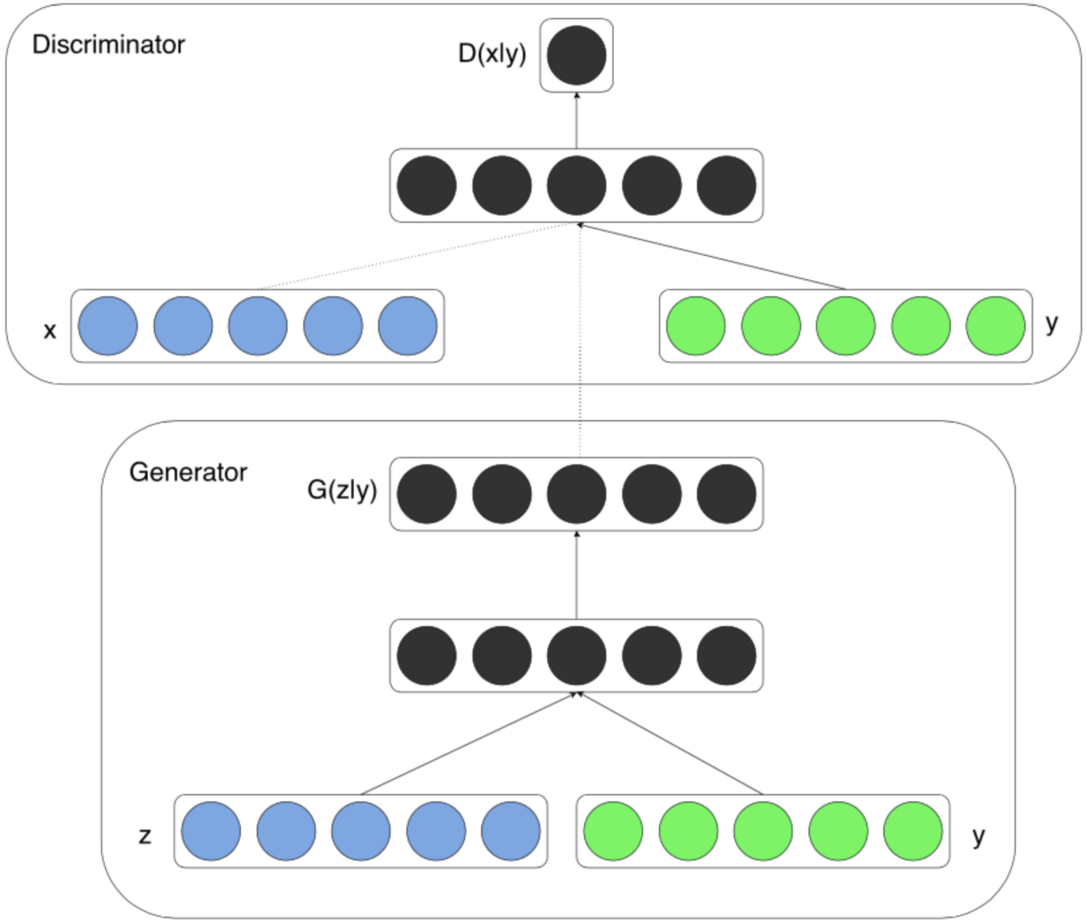


(a) Azimuth (pose) (b) Elevation

(c) Lighting (d) Wide or Narrow

GAN spinoffs

- Conditional GANs
  - Standard GANs have **no encoder**!
- Actor-Critic
  - Related to Reinforcement Learning



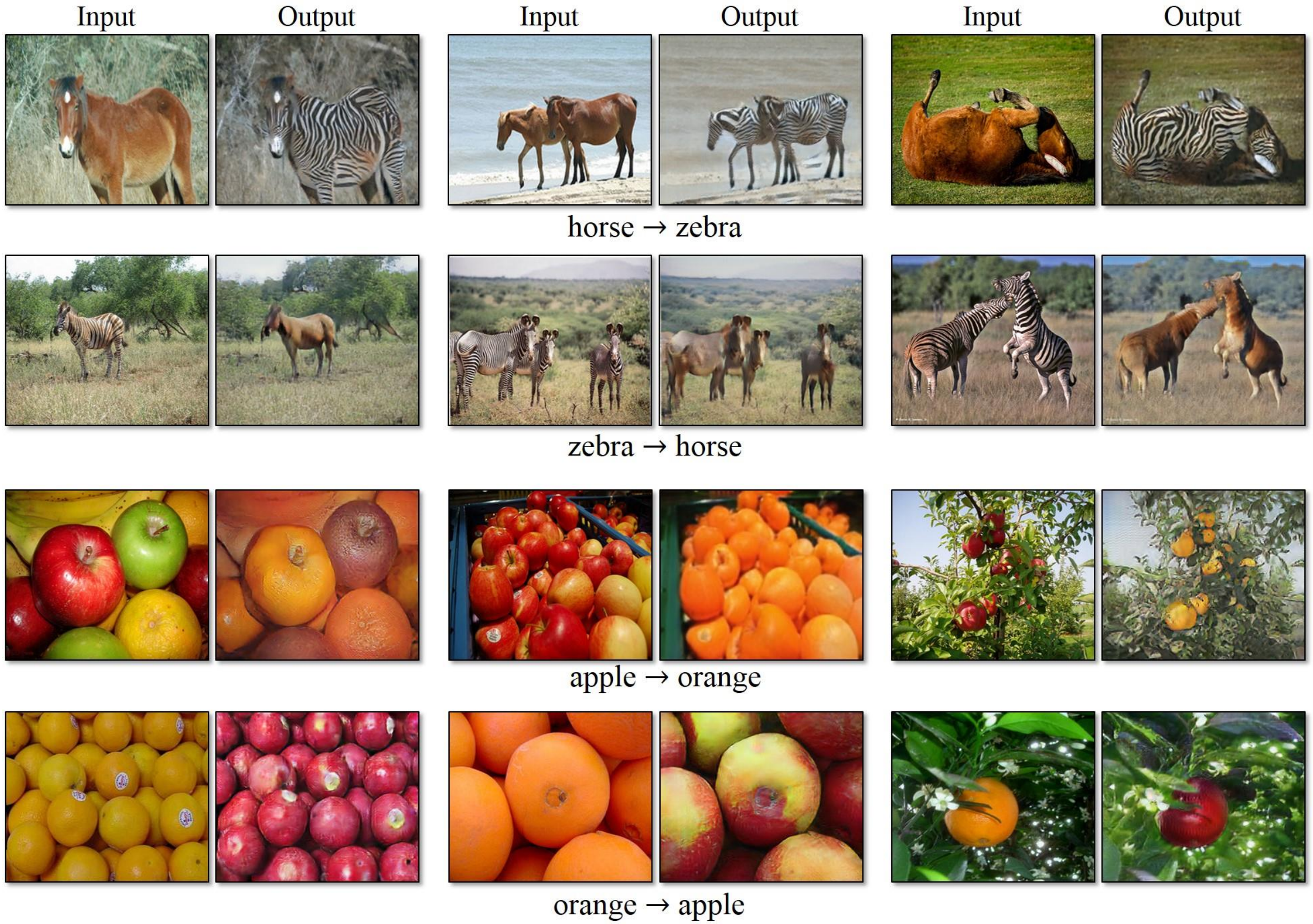
Conditional GAN

Connections to Reinforcement Learning

- GANs interpreted as actor-critic [Pfau and Vinyals, 2016]
- GANs as inverse reinforcement learning [Finn et al., 2016]
- GANs for imitation learning [Ho and Ermin 2016]

Application: Image to Image translation

Application: Style transfer



Application: Face generation

- https://www.youtube.com/watch?v=XOxxPcy5Gr4

Summary

- GANs are generative models using supervised learning to approximate an intractable cost function
- GANs can simulate many cost functions, including max likelihood
- Finding Nash equilibria in high-dimensional, continuous, non-convex games is an important open research problem
- GAN research is in its infancy, most works published only in 2016. Not mature enough yet, but very compelling results